<table>
<thead>
<tr>
<th>Time</th>
<th>Components</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-15 minutes</td>
<td>Number Talks</td>
<td>Short, daily fluency routine that engages students in meaningful conversations around purposefully crafted computation problems that are solved using number relationships and the structure of numbers. Students are asked to communicate their thinking when presenting and justifying solutions to problems they solve mentally while the teacher records their ideas with mathematical precision. These exchanges lead to the development of more accurate, efficient, and flexible strategies.</td>
</tr>
<tr>
<td>5 minutes</td>
<td>Opening: Hook/Coherence Connection</td>
<td>The teacher will engage students to create interest for the whole group lesson or review prerequisite standards to prepare students to make explicit connections that will allow students to apply and extend previous learning when interacting with the lesson’s grade-level content.</td>
</tr>
<tr>
<td>15 minutes</td>
<td>Whole Group: Mini Lesson/Guided Practice</td>
<td>Used prior to small group to introduce/practice new knowledge and skills or after small group to refine/practice strategies discovered by students. The lesson focuses on the depth of grade-level cluster(s), grade-level content standard(s), or part(s) thereof, intentionally targeting the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed. During this time, the teacher makes the mathematics of the lesson explicit using clear and correct explanations, representations, tasks, and/or examples. The teacher provides opportunities for all students to work with and practice grade-level problems and exercises, deliberately checking for understanding throughout the lesson and adapting the lesson according to student understanding. The teacher poses high-quality questions and problems that prompt students to share their developing thinking about the content of the lesson. Class created anchor charts are constructed by strategically adding key concepts throughout the topic’s lessons.</td>
</tr>
<tr>
<td>30-40 minutes</td>
<td>Small Collaborative Groups/Independent Practice</td>
<td>The teacher encourages reasoning and problem solving by posing challenging problems that offer opportunities for student choice of appropriate tools and promote productive struggle. Students work in small, flexible collaborative groups to engage in mathematical tasks while the teacher circulates and asks questions to elicit thinking, providing support or extensions as needed. The teacher asks students to explain and justify work, connecting and developing students’ informal language to precise mathematical language appropriate to their grade, and provides feedback that helps students revise initial work. The teacher makes observations to select and sequence appropriate strategies for students to share during the class discussion.</td>
</tr>
<tr>
<td>5 minutes</td>
<td>Closure: Summarize</td>
<td>The teacher strengthens all students’ understanding of the content by strategically sharing a variety of students’ representations and solution methods. The teacher facilitates the summary of the mathematics with references to student work and by creating the conditions for student conversations where students are encouraged to talk about each other’s thinking in order to reinforce the purpose of the lesson.</td>
</tr>
<tr>
<td>Units</td>
<td>Topics</td>
<td>Standards</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>1</td>
<td>Applying place value concepts in whole number addition and subtraction (12 days)</td>
<td>4.NBT.1.1, 4.NBT.1.2, 4.NBT.1.3, 4.NBT.1.4 (not fluently)</td>
</tr>
<tr>
<td>2</td>
<td>Exploring multiples and factors (11 days)</td>
<td>4.OA.2.4, 4.OA.3.5 (only number patterns)</td>
</tr>
<tr>
<td>3</td>
<td>Using multiplication and division strategies with larger numbers (23 days)</td>
<td>4.OA.1.3 (only multiplication and division), 4.NBT.2.5 (only arrays and area models), 4.NBT.2.6 (only arrays and area models), 4.MD.1.3, 4.OA.1.a and 4.OA.1.b (Number Talks)</td>
</tr>
<tr>
<td>4</td>
<td>Decomposing and composing fractions for addition and subtraction (8 days)</td>
<td>4.NF.2.3 (a, b) (no mixed numbers)</td>
</tr>
<tr>
<td>5</td>
<td>Understanding fraction equivalence and comparison (12 days)</td>
<td>4.NF.1.1, 4.NF.1.2</td>
</tr>
<tr>
<td>6</td>
<td>Introducing measurement conversions (11 days)</td>
<td>4.OA.1.1, 4.NBT.1.1, 4.MD.1.1</td>
</tr>
<tr>
<td>7</td>
<td>Solving problems using multiplicative comparison (12 days)</td>
<td>4.OA.1.2, 4.NBT.1.3, 4.MD.1.2 (only multiplication and division, only whole numbers), 4.OA.1.a and 4.OA.1.b (Number Talks)</td>
</tr>
<tr>
<td>8</td>
<td>Solving measurement problems using the four operations (12 days)</td>
<td>4.OA.1.3, 4.NBT.2.4 (not fluently), 4.MD.1.2 (only whole numbers), 4.OA.1.a and 4.OA.1.b (Number Talks), 4.NF.2.3 (c, d), 4.MD.2.4</td>
</tr>
<tr>
<td>9</td>
<td>Solving addition and subtraction problems involving fractions and mixed numbers (10 days)</td>
<td>4.MD.3.5, 4.MD.3.6, 4.G.1.1 (not parallel, perpendicular, or 2-D figures)</td>
</tr>
<tr>
<td>10</td>
<td>Angle measurement (12 days)</td>
<td>4.OA.1.1, 4.NF.2.4</td>
</tr>
<tr>
<td>11</td>
<td>Multiplying fractions by whole numbers (13 days)</td>
<td>4.OA.1.1, 4.NF.2.4</td>
</tr>
<tr>
<td>13</td>
<td>Recognizing and analyzing attributes of 2-dimensional shapes (12 days)</td>
<td>4.OA.1.1, 4.OA.1.3, 4.NBT.2.4, 4.NBT.2.5, 4.NBT.2.6</td>
</tr>
</tbody>
</table>
# Unit 1

### Topic 1: Applying place value concepts in whole number addition and subtraction

The focus of this topic is to provide students time to develop and practice efficient addition and subtraction of multi-digit whole numbers while developing place value concepts. This topic also includes rounding, which provides students with another strategy to judge the reasonableness of their answers in addition and subtraction situations. In Grade 3 (3.NBT.1.1), students used place value understanding to round whole numbers to the nearest 10 or 100, which is extended this year to rounding multi-digit whole numbers to any place.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <em>For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.</em></td>
<td>MAFS.4.NBT.1.1 base-ten numeral compose decompose digit expanded form greater than symbol (&gt;) less than symbol (&lt;) multi-digit number names standard algorithm value</td>
</tr>
</tbody>
</table>

**Students will:**
- **demonstrate** with models that in a multi-digit whole number, a digit in one place represents ten times the value of the place to its right.

<table>
<thead>
<tr>
<th></th>
<th>thousands</th>
<th>hundreds</th>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.g.</td>
<td>3,000</td>
<td>300</td>
<td>30</td>
<td>3</td>
</tr>
</tbody>
</table>

- **explain** that a digit in one place is worth ten times the value of the place to its right.

E.g., 10 × 540 = 5,400; the 5 in 5,400 represents 5 thousands which is 10 times as much as the 5 hundreds in 540.

**NOTE:** Students will not compare digits across more than 1 place value.

**NOTE:** While students may discover the pattern of additional zeros at the end of a product, they need to be able to justify this pattern in terms of place value. When you multiply a number by 10, the value becomes 10 times greater, so every digit shifts one place to the left, or in 10 x 4, ten groups of 4 ones is 40 ones, which is equivalent to 4 tens, or 40.
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

**MAFS.4.NBT.1.2**

**Students will:**

**NOTE:** In grade 2, students learned to read and write numbers to 1,000 using base-ten numerals, number names, and expanded form (2.NBT.1.3). There are no explicit place value standards in grade 3, however students continue to use their place value understanding from grade 2 to add, subtract, and round within 1,000 (3.NBT.1.1 and 1.2). This will be the first formal exposure students have to numbers greater than 1,000. For this reason, special attention must be focused on numbers between 1,000 and 1,000,000 in grade 4.

- **read** and **write** multi-digit numbers through one million in base-ten numerals, number names, and expanded form.

**NOTE:** Expanded form of 285 can be 200 + 80 + 5. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens and 5 ones (necessary for fluent regrouping using the standard algorithm).

- **compare** two multi-digit numbers to 1,000,000 using place value and record the comparison numerically using the following symbols: <, >, or =.

**MAFS.4.NBT.1.3**

**Students will:**

- **understand** that the purpose of rounding to estimate before adding or subtracting is to make mental math easier and to check the reasonableness of an answer.

- **use** place value understanding to round multi-digit whole numbers (between 1,000 and 1,000,000) to any place.

**E.g.,**

Represent the number 38,450 on a number line.

Possible student response:
On the number line, 38,450 is between 30,000 and 40,000. The number 38,450 is closer to 40,000 than to 30,000. So, 38,450 rounded to its nearest ten thousands is 40,000.
### Fluent addition and subtraction of multi-digit whole numbers using the standard algorithm

<table>
<thead>
<tr>
<th>Students will:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• apply an understanding of addition and subtraction, place value, and flexibility with multiple strategies to use the standard algorithms for addition and subtraction of whole numbers with solutions greater than 1,000 and within 1,000,000.</td>
<td><strong>MAFS.4.NBT.2.4</strong></td>
</tr>
</tbody>
</table>

**NOTE:** Computational fluency is defined as accuracy, efficiency, and flexibility. Grade 4 is the first grade-level in which students are expected to be fluent with the standard algorithm to add and subtract. Fluency with the standard algorithm is an end of the year grade level expectation.

6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MAFS.K12.MP.6.1</strong></td>
<td><strong>MAFS.K12.MP.8.1</strong></td>
</tr>
</tbody>
</table>

### Topic Comments:

- **NBT.1.1** will be revisited in topic 6 connected to conversions within the metric system of measurement.
- **NBT.1.3** will be revisited in topic 7 with multiplication and division as a context.
- **NBT.2.4** will be revisited in topic 8 and finalized in topic 14 for fluency in addition and subtraction of multi-digit whole numbers.

Students use the structure of the base-ten system to generalize their strategies and to discuss reasonableness of their computations and work towards fluency (**MP.6, MP.8**).
### Topic 2: Exploring multiples and factors

In this topic students develop an understanding of multiples and factors, applying their understanding of multiplication from the previous year (Fluently multiplying and dividing within 100 and knowing all products of two one-digit numbers from memory (3.OA.3.7)). This understanding lays a strong foundation for generalizing strategies learned in previous grades to develop, discuss, and use efficient, accurate, and generalizable computational strategies involving multi-digit numbers. These concepts and the terms “prime” and “composite” are new to Grade 4, so they are introduced early in the year to give students ample time to develop and apply this understanding.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
</table>
| Investigate factors and multiples.  
  a. Find all factor pairs for a whole number in the range 1–100.  
  b. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.  
  c. Determine whether a given whole number in the range 1–100 is prime or composite. | MAFS.4.OA.2.4 |

**NOTE:** This standard has been amended in Florida.

### Students will:
- **determine** factor pairs of whole numbers in the range 1-100.
  
  E.g.,

<table>
<thead>
<tr>
<th>24</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

- **recognize** that a whole number is a multiple of each of its factors.
- **determine** whether a given whole number in the range 1-100 is a multiple of a given one-digit number.
- **determine** if a number in the range 1-100 is prime or composite.

**E.g.,**

<table>
<thead>
<tr>
<th>1 x 7 = 7</th>
<th>9 x 1 = 9</th>
<th>3 x 3 = 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>x x x x x x x</td>
<td>x x x x x x x x x</td>
<td>x x x x x x x x x</td>
</tr>
</tbody>
</table>

7 is prime because it has only one and itself as factors. Because 1 x 7 is commutative, one and seven are 7’s only factor pair.

9 is composite because it has two or more factor pairs. Because 1 x 9 is commutative, one and nine are 1 factor pair and, three and three are a second factor pair of 9.
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

**Students will:**
- **generate** a number pattern that follows a given one- or two-step rule.
  
  E.g.,
  Rule: Multiply by 2 and then subtract 4.
  The first number in the pattern is 6.
  Show the next three numbers in the pattern.
  Answer: 8, 12, 20

- **identify** features in a number pattern after following a given one- or two-step rule.
  
  E.g.,

<table>
<thead>
<tr>
<th>Rule</th>
<th>Pattern</th>
<th>Feature(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with 4 and add 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4, 9, 14, 19, 24, 29, 34, 39, ...</td>
<td>The numbers alternate ending in a 4 or 9.</td>
<td></td>
</tr>
<tr>
<td>Start with 1 and multiply by 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1, 5, 25, 125, 625, 3125, ...</td>
<td>After the starting number, all numbers that follow end in 5. All the numbers that have resulted are odd.</td>
<td></td>
</tr>
<tr>
<td>Start with 100 and subtract 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100, 97, 94, 91, 88, 85, 82, 79, 76, 73, ...</td>
<td>The numbers alternate from even to odd. All digits from 0-9 eventually end up in the ones place.</td>
<td></td>
</tr>
</tbody>
</table>

3. Construct viable arguments and critique the reasoning of others.  
7. Look for and make use of structure.  

**Topic Comments:**
While working on OA.2.4, students use manipulatives to determine whether a number is prime or composite. Although there are shape patterns in arrays, the focus of this topic is number patterns. OA.3.5 is repeated in topic 13, where the focus will be on identifying shape patterns.

The focus of this topic is for students to search systematically to find factor pairs and examine various patterns in order to interpret the relationship between multiples and their factors. Students use prior knowledge of the concept and language of division to discuss the structure of multiples and factors (MP.3, MP.7).
**Topic 3: Using multiplication and division strategies with larger numbers**

In this topic, students continue using computational and problem-solving strategies, with a focus on building conceptual understanding of multiplication of greater numbers and division with remainders. Area and perimeter of rectangles provide one context for developing such understanding.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td>
<td>MAFS.4.OA.1.3 area model comparative relational thinking distributive property dividend divisor factor formula mental computation reasonableness rectangular array remainder quotient unknown factor unknown quantity variable</td>
</tr>
</tbody>
</table>

**Students will:**
- **solve** multi-step word problems (up to 3 steps) that involve multiplication and/or division using strategies for this grade level (E.g., rectangular arrays, area models, properties of operations, etc.).
- **represent** a multi-step word problem using equations involving a variable represented by a letter for the unknown number.
- **interpret** remainders that result from multi-step word problems.
- **assess** the reasonableness of answers to multi-step word problems using estimation strategies and mental computation prior to calculation.

**NOTE:** The expectations for multi-step word problems in this topic include multiplication of 2-digit by 1-digit and division of 2-digit by 1-digit.

*Students will write remainders as fractions in Grade 5*
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Students will:**
- use manipulatives or drawings of rectangular arrays and/or array models to solve and explain multi-digit multiplication problems that extend to 4-digit by 1-digit or up to 2-digit by 2-digit.

**E.g.,**

<table>
<thead>
<tr>
<th>Rectangular Array (Concrete)</th>
<th>Rectangular Array (Representational)</th>
<th>Area Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 × 21 = 63</td>
<td>13 × 13 = 169</td>
<td>34 × 28 = 952</td>
</tr>
</tbody>
</table>

**NOTE:** In this topic students develop a conceptual understanding by connecting the multiplication process to the context of finding Area.

**NOTE:** Use of the standard algorithm for multiplication is a Grade 5 standard. Although these strategies are precursors to the standard algorithm, students should not be moved too quickly to the algorithm, as the standard algorithm is not the only end goal. Perhaps more importantly, students must become flexible in their use of place value and properties of operations when multiplying in Grade 4 in order to be successful later in Algebra when using the Distributive Property.

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Students will:**
- use manipulatives or drawings of rectangular arrays and/or area models to solve and explain division problems that involve the division of a multi-digit dividend (with up to four digits) by a one-digit divisor.

**E.g.,**

<table>
<thead>
<tr>
<th>Rectangular Array (Concrete)</th>
<th>Rectangular Array (Representational)</th>
<th>Area Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>63 ÷ 3 = 21</td>
<td>169 ÷ 13 = 13</td>
<td>952 ÷ 34 = 28</td>
</tr>
</tbody>
</table>

**NOTE:** Use of the standard algorithm for division is a Grade 6 standard. In this topic students develop a conceptual understanding by connecting the division process to the context of Area with an unknown side length.
Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.  

**MAFS.4.MD.1.3**

**Students will:**
- apply the perimeter formula \( P = 2l + 2w \) in real world and mathematical situations.
- apply the area formula \( A = l \times w \) in real world and mathematical situations.
- solve for missing dimensions of rectangles when provided with the perimeter and/or area.
- solve problems involving area of a composite figure composed of rectangles.

**Note:** Students need to be provided with the Grade 4 FSA Mathematics Reference Sheet in order to practice using the tool. The Reference Sheet presents the Area formula as \( A = lw \), so students must be introduced to the convention that when two unknowns are together without an operational symbol it is another way to represent multiplication.

Exponential notation for unit abbreviations (e.g., \( cm^2 \)) is not introduced until Grade 5.

Determine whether an equation is true or false by using comparative relational thinking. For example, without adding 60 and 24, determine whether the equation \( 60 + 24 = 57 + 27 \) is true or false.

**MAFS.4.OA.1.a**

**Note:** This standard has been added in Florida.

**Students will:**

**Note:** Due to this standard’s emphasis on mental math, it will be introduced and then embedded through the use of Number Talks.
- determine if a given equation is true or false by comparing, composing, and/or decomposing the numbers without solving.

**E.g.,**

```latex
\begin{align*}
\text{Teacher:} & \quad \text{Use comparative relational thinking to determine if the following equation is true or false.} \\
& \quad 8 + 4 = 7 + 5 \\
\text{Student:} & \quad \text{True. I saw that the 5 over here [pointing to the 5 in the number sentence] was one more than the 4 over here [pointing to the 4 in the number sentence], since 7 is one less than 8 the equation is true.} \\
\text{Teacher:} & \quad \text{Let's try another one. How about } 57 + 96 = 59 + 84? \\
\text{Student:} & \quad \text{That's easy. True. It's just like the other one. 59 is two more than 57, and 84 is two less than 86.}
\end{align*}
```

**E.g.,**

```latex
\begin{align*}
\text{Use comparative relational thinking to determine if the following equation is true or false.} \\
8 \times 76 = 8 \times 70 + 8 \times 5 & \quad (\text{Possible response: False. 8 groups of 70 and 8 groups of 5 has the same value as 8 groups of 75, which is less than 8 groups of 76})
\end{align*}
```

**Note:** Whole number equations are limited to: addition and subtraction within 1,000, multiplication of 2-digit by 1-digit or division of 2-digit by 1-digit.
Determine the unknown whole number in an equation relating four whole numbers using comparative relational thinking. For example, solve $76 + 9 = n + 5$ for $n$ by arguing that nine is four more than five, so the unknown number must be four greater than 76.  

*NOTE: This standard has been added in Florida.*

### Students will:

**NOTE:** Due to this standard’s emphasis on mental math, it will be introduced and then embedded through the use of Number Talks.

- **compare** the two sides of an equation to **determine** the unknown value without solving.

  E.g., Use comparative relational thinking to determine if the following equation is true or false.  
  
  $27 + n = 25 + 42$  
  (Possible response: “25 is 2 less than 27, so $n$ must be 2 less than 42.”)

  E.g., Use comparative relational thinking to determine if the following equation is true or false.  
  
  $3 \times 50 + 3 \times 9 = 3 \times n$  
  (Possible response: $n=59$. This shows the Distributive Property. 3 groups of 50 and 3 groups of 9 has the same value as 3 groups of 59)

  **NOTE:** Whole number equations are limited to: addition and subtraction within 1,000, multiplication of 2-digit by 1-digit or division of 2-digit by 1-digit.

<table>
<thead>
<tr>
<th>Topic Comments:</th>
</tr>
</thead>
</table>
| **OA.1.3** is the first time students are expected to interpret remainders based upon the context. All four operations will be addressed in topic 8, and the standard will be finalized in topic 14.  
**MD.1.3** provides the context of area and perimeter of rectangles to use for problem solving. Students are first introduced to formulas in this topic and make sense of the formulas using their prior work from Grade 3 (3.MD.3.5, 3.MD.3.6, 3.MD.3.7, 3.MD.4.8) with area and perimeter.  
**NBT.2.5 & NBT.2.6** will be finalized in topic 14 to include equations and strategies such as partial products and partial quotients. The use of these strategies is a step toward the standard algorithms for multiplication and division. Students are still building conceptual understanding of these two operations in Grade 4 and the standard algorithms are not used until Grades 5 and 6. This being said, these strategies are reserved for later in the scope and sequence as a bridge to the algorithms, after conceptual understanding has been addressed throughout the year through the use of arrays and area models. 
Students make sense of multi-step problems (MP.1) and reason about how the formulas connect to the context (MP.2). The use of generalized strategies and formulas provides an opportunity to investigate and use regularity in repeated reasoning (MP.8). |
## Unit 2

### Topic 4: Decomposing and composing fractions for addition and subtraction

In this topic, students extend their prior knowledge of unit fractions with denominators of 2, 3, 4, 6, and 8 from Grade 3 (3.NF.1.1) to include denominators of 5, 10, 12, and 100. In Grade 4, they use their understanding of partitioning to find unit fractions to compose and decompose fractions in order to add fractions with like denominators. This is foundational for further work with fractions later in the year, such as comparing fractions and multiplying fractions by a whole number.

### Standards

Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples:

\[
\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}; \quad \frac{2}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}
\]

### Academic Language

- decompose
- denominator
- equation
- equivalent fraction
- numerator
- sum
- unit fraction
- whole

**Students will:**

- **demonstrate** with concrete and visual models (i.e., number lines, rectangles, squares, and circles) that adding fractions referring to the same size whole is joining parts of the wholes.
- **demonstrate** with concrete and visual models that subtracting fractions referring to the same size whole is separating parts of the wholes.

**NOTE:** Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- **decompose** a fraction (less than or greater than 1) into a sum of fractions with the same denominator in more than one way using visual models.
- **record** the decomposition of a fraction by an equation and **justify** the decomposition.

**E.g.**

\[
\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \quad \text{or} \quad \frac{3}{8} = \frac{1}{8} + \frac{2}{8}
\]

**E.g.**

\[
\frac{9}{8} = \frac{8}{8} + \frac{1}{8} \quad \text{or} \quad \frac{15}{6} = \frac{6}{6} + \frac{6}{6} + \frac{3}{6}
\]
7. Look for and make use of structure.  

<table>
<thead>
<tr>
<th>Topic Comments:</th>
<th>MAFS.K12.MP.7.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF.2.3 will be finalized in topic 9 to address adding and subtracting mixed numbers.</td>
<td></td>
</tr>
<tr>
<td>Students use their previous experiences with composing and decomposing whole numbers to understand the structure of fraction composition (MP 7).</td>
<td></td>
</tr>
</tbody>
</table>
### Topic 5: Understanding fraction equivalence and comparison

In this topic students develop an understanding of fraction equivalence and various methods for comparing fractions. Students should understand that when comparing fractions, it is not always necessary to generate equivalent fractions. Other methods, such as comparing fractions to a benchmark, can be used to discuss relative sizes. The justification of comparing or generating equivalent fractions using visual models is an emphasis of this topic.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain why a fraction ( \frac{a}{b} ) is equivalent to a fraction ( \frac{n \times a}{n \times b} ) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</td>
<td>benchmark fraction common denominator common numerator denominator equivalent fractions fraction greater than 1 greater than symbol (&gt;) less than symbol (&lt;) numerator unit fraction whole</td>
</tr>
</tbody>
</table>

**Students will:**
- **explain**, using visual representation, how and why fractions can be equivalent even though the number and size of the parts are not the same.
- **recognize** and **generate** equivalent fractions by partitioning number lines, rectangles, squares, and circles using visual models.

E.g.,

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \frac{1}{2} \quad \frac{2}{4} \quad \frac{4}{8} \]

**NOTE:** Denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, 100.

**NOTE:** In Grade 4 students recognize and generate equivalent fractions using visual models, in Grade 5 (after learning how to multiply a fraction by a fraction) students will extend this understanding of equivalency to understand that multiplying by a fraction equivalent to 1 (e.g. \( \frac{4}{4} \)) will result in an equivalent fraction.
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

**Students will:**

- **explain** that fractions can only be compared when they refer to the same sized whole (e.g., $\frac{1}{2}$ of a small pizza is not the same size as $\frac{1}{2}$ of a large pizza).

  **NOTE:** Denominators limited to 2, 3, 4, 5, 6, 8, 10, 12, 100.

- **compare** two fractions with different numerators and different denominators by:
  - using benchmark fractions (e.g., $\frac{5}{8}$ is close to $\frac{1}{2}$ and $\frac{1}{10}$ is close to 0, therefore $\frac{5}{8} > \frac{1}{10}$).
  - reasoning about their size or location on a number line.
  - using visual models to create fractions with common numerators or common denominators.

- **record** the results of comparisons with the symbols $<$, $>$ or $=$.

- **justify** the conclusions of comparisons.

  **NOTE:** Fractions may be greater than 1 (e.g., $\frac{6}{5}$).
  It is not necessary for students to find the least common denominator (LCD).

3. Construct viable arguments and critique the reasoning of others.  
5. Use appropriate tools strategically.

**Topic Comments:**

Students justify their methods for generating equivalent fractions and comparing fractions by using their conceptual understanding and models (MP.3, MP.5).
## Topic 6: Introducing measurement conversions

In this topic students build a conceptual understanding of the relative sizes of units of measure within a single system of measurement. Measurement conversions are used to introduce multiplication as a comparison. The concepts in this topic are foundational for the concepts in topic 7 and topic 8.

### Standards

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that $35$ is $5$ times as many as $7$ and $7$ times as many as $5$. Represent verbal statements of multiplicative comparisons as multiplication equations.</td>
<td>MAFS.4.OA.1.1</td>
</tr>
</tbody>
</table>

**Students will:**

- **interpret** a basic multiplication equation as a comparison (e.g., if $a = n \times b$, then $a$ is $n$ times as much as $b$).
- **represent** verbal statements of multiplicative comparison as multiplication equations.

**NOTE:** Statements for comparative language: times as tall as, times as long as, times as heavy as, times as much as, etc. (rather than times taller, times longer, times heavier, etc.).

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.  

**Students will:**

- **explain** that a digit in one place is worth ten times the value of the place to its right by connecting to measurement conversions within the metric system.

E.g.,

<table>
<thead>
<tr>
<th>thousand</th>
<th>hundred</th>
<th>ten</th>
<th>ONES</th>
<th>tenth</th>
<th>hundredth</th>
<th>thousandth</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilo-</td>
<td>hecto-</td>
<td>deca-</td>
<td>METER</td>
<td>deci-</td>
<td>centi-</td>
<td>milli-</td>
</tr>
</tbody>
</table>

= 1000 ×  

= 100 ×
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; L, mL; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …

Students will:

NOTE: Students need to be provided with the Grade 4 FSA Mathematics Reference Sheet in order to practice using the tool.

- know relative size of measurement units within one system.
- convert larger units of measure into smaller equivalent units (required units are listed on Grade 4 FSA Mathematics Reference Sheet).
- complete a two-column table (function table) showing measurement equivalents and relate measurement conversions to multiplicative comparisons (e.g., 1 yard is 3 times as long as 1 foot).

E.g.,

<table>
<thead>
<tr>
<th>lb</th>
<th>oz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ft</th>
<th>in</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>kg</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
</tr>
</tbody>
</table>

2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

Topic Comments:

OA.1.1 is repeated in topic 11, in which the focus is on multiplication of fractions by whole numbers.
NBT.1.1 was addressed in topic 1, in which the focus was on addition and subtraction. In this topic, metric measurement provides an opportunity to deepen the students’ understanding of place value in relation to multiples of 10.
MD.1.1 introduces units of measure new to Grade 4.

In this topic students look for patterns in different measurement systems (MP.2, MP.7) and discuss precisely how many times larger one unit is than another (MP.6)
**Topic 7: Solving problems using multiplicative comparison**

In this topic students are introduced to multiplicative compare problems, extending their conceptual work with multiplicative comparison from topic 6. For students to develop this concept, they must be provided rich problem situations that encourage them to make sense of the relationships among the quantities involved, model the situation, and check their solution using a different method.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</td>
<td>additive comparison balanced equation comparative relational thinking intervals of time linear models multiplicative comparison unknown number</td>
</tr>
</tbody>
</table>

**Students will:**

- **distinguish** multiplicative comparison (e.g., Tonya has 3 times as many cousins as Matthew) from additive comparison (e.g., Tonya has 3 more cousins than Matthew).

  **NOTE:** In an additive comparison, the underlying question is what amount would be added to one quantity in order to result in the other. In a multiplicative comparison, the underlying question is what factor would multiply one quantity in order to result in the other.

- solve word problems involving multiplicative comparison (situations may include 2-digit by 1-digit expressions).

  **E.g., Common Multiplication and Division Table**

<table>
<thead>
<tr>
<th>Larger Unknown</th>
<th>Smaller Unknown</th>
<th>Multiplier Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compare</strong></td>
<td>A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</td>
<td><strong>Measurement example.</strong> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</td>
</tr>
<tr>
<td>A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does the blue hat cost?</td>
<td><strong>Measurement example.</strong> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</td>
<td></td>
</tr>
<tr>
<td><strong>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?</strong></td>
<td><strong>Measurement example.</strong> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</td>
<td></td>
</tr>
</tbody>
</table>

**E.g., A red hat costs $18 and a blue hat costs $6. How many times as much as the blue hat does the red hat cost?**

<table>
<thead>
<tr>
<th></th>
<th>blue hat</th>
<th>red hat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6</td>
<td>$6</td>
<td>$6</td>
</tr>
<tr>
<td>$6</td>
<td>$6</td>
<td>$18</td>
</tr>
</tbody>
</table>

The red hat costs 3 times as much as the blue hat.
<table>
<thead>
<tr>
<th>Use place value understanding to round multi-digit whole numbers to any place.</th>
<th>MAFS.4.NBT.1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Students will:</strong></td>
<td></td>
</tr>
<tr>
<td>• understand that the purpose of rounding to estimate before multiplying or dividing is to make mental math easier and to check the reasonableness of an answer.</td>
<td></td>
</tr>
<tr>
<td>• use place value understanding to round multi-digit whole numbers to any place between 1,000 and 1,000,000.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Use the four operations to solve word problems involving distances, intervals of time, and money, including problems involving simple fractions or decimals. Represent fractional quantities of distance and intervals of time using linear models.</th>
<th>MAFS.4.MD.1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NOTE:</strong> This is an amended Florida Standard.</td>
<td></td>
</tr>
<tr>
<td><strong>Students will:</strong></td>
<td></td>
</tr>
<tr>
<td>• use multiplication and division to solve multiplicative comparison word problems involving distances (i.e., inch, feet, yard, mile; millimeter, centimeter, meter, kilometer), including problems that require expressing measurements given in a larger unit in terms of a smaller unit, involving whole numbers only.</td>
<td></td>
</tr>
<tr>
<td>• use multiplication and division to solve multiplicative comparison word problems involving intervals of time including whole numbers only and represent the intervals of time using linear models.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E.g., Henry and James started working on a school project at 3:00. Henry spent 15 minutes working on his project and James spent 3 times as long working on his project. What time did James finish working on his project?</th>
<th></th>
</tr>
</thead>
</table>

| Use the four operations to solve word problems involving distances, intervals of time, and money, including problems involving simple fractions or decimals. Represent fractional quantities of distance and intervals of time using linear models. | |

<table>
<thead>
<tr>
<th>Determine whether an equation is true or false by using comparative relational thinking. For example, without adding 60 and 24, determine whether the equation (60 + 24 = 57 + 27) is true or false.</th>
<th>MAFS.4.OA.1.a</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NOTE:</strong> This standard has been added in Florida.</td>
<td></td>
</tr>
<tr>
<td><strong>Students will:</strong></td>
<td></td>
</tr>
<tr>
<td>• determine if a given equation is true or false by comparing, composing, and/or decomposing the numbers without solving.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>NOTE:</strong> Due to this standard’s emphasis on mental math, it will be embedded through the use of Number Talks.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• determine if a given equation is true or false by comparing, composing, and/or decomposing the numbers without solving.</td>
<td></td>
</tr>
<tr>
<td><strong>NOTE:</strong> Equations in this topic may include: Multiplication of 2-digit by 1-digit or division of 2-digit by 1-digit.</td>
<td></td>
</tr>
</tbody>
</table>
Determine the unknown whole number in an equation relating four whole numbers using comparative relational thinking. For example, solve $76 + 9 = n + 5$ for $n$ by arguing that nine is four more than five, so the unknown number must be four greater than 76.

*NOTE: This standard has been added in Florida.*

<table>
<thead>
<tr>
<th>Students will:</th>
<th>MAFS.4.OA.1.b</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTE: Due to this standard’s emphasis on mental math, it will be embedded through the use of Number Talks.</td>
<td></td>
</tr>
<tr>
<td>• compare the two sides of an equation to determine the unknown value without solving.</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Equations in this topic may include: Multiplication of 2-digit by 1-digit or division of 2-digit by 1-digit.

1. Make sense of problems and persevere in solving them.  

<table>
<thead>
<tr>
<th>Topic Comments:</th>
<th>MAFS.K12.MP.1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>OA.1.2 is also addressed in topic 14 because of the time it takes to master the concepts and its importance to future mathematics.</td>
<td></td>
</tr>
<tr>
<td>NBT.1.3 was addressed in topic 1 with a focus on addition and subtraction. It is finalized in this topic, to focus on multiplication and division.</td>
<td></td>
</tr>
<tr>
<td>MD.1.2 is used as a context for multiplicative compare problems with whole numbers only. This standard is revisited in topic 8 to include addition and subtraction, and finalized in topic 12 with decimal fractions.</td>
<td></td>
</tr>
<tr>
<td>Students use charts and diagrams to explain their own methods as well make sense of approaches taken by others (MP.1).</td>
<td></td>
</tr>
</tbody>
</table>
**Unit 3**

**Topic 8: Solving measurement problems using the four operations**

In this topic students combine competencies from different domains to solve measurement problems using the four operations. Measurement is included in this topic to provide a context for problem solving. All of the problem types in the Common Addition and Subtraction and Multiplication and Division Tables (see pg. 43 and 44) should be addressed in this topic.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td>
<td>MAFS.4.OA.1.3</td>
</tr>
</tbody>
</table>

**Students will:**

- **solve** multi-step word problems (i.e., up to 3 steps) that involve the four operations using strategies for this grade level (e.g., rectangular arrays, area models, properties of operations, standard algorithm for addition and subtraction, etc.).
- **represent** a multi-step word problem using equations involving a variable represented by a letter for the unknown number.
- **interpret** remainders that result from multi-step word problems.
- **assess** the reasonableness of answers to multi-step word problems using estimation strategies and mental computation prior to calculation.

**NOTE:** The expectations for multi-step word problems include addition and subtraction within 1,000, multiplication of 2-digit by 1-digit, and division of 2-digit by 1-digit.

- Fluently add and subtract multi-digit whole numbers using the standard algorithm. | MAFS.4.NBT.2.4 |

**Students will:**

- **apply** an understanding of addition and subtraction, place value, and flexibility with multiple strategies to use the standard algorithms for addition and subtraction.

**NOTE:** Addition expressions may contain up to three addends.

- Use the four operations to solve word problems involving distances, intervals of time, and money, including problems involving simple fractions or decimals. Represent fractional quantities of distance and intervals of time using linear models.  | MAFS.4.MD.1.2 |

**Students will:**

- **use** the four operations to solve word problems involving distances (i.e., inch, feet, yard, mile; millimeter, centimeter, meter, kilometer), including problems that require expressing measurements given in a larger unit in terms of a smaller unit, involving whole numbers only.
- **use** the four operations to solve word problems involving intervals of time including whole numbers only and **represent** the intervals of time using linear models.
- **use** the four operations to solve word problems involving money including whole numbers only.

**NOTE:** All of the problem types in the Common Addition and Subtraction and Multiplication and Division Tables (see pg. 43 and 44) should be addressed in this topic.
Determine whether an equation is true or false by using comparative relational thinking. For example, without adding 60 and 24, determine whether the equation $60 + 24 = 57 + 27$ is true or false. **MAFS.4.OA.1.a**

### Students will:
- **NOTE:** Due to this standard’s emphasis on mental math, it will be embedded through the use of Number Talks.
- **determine** if a given equation is true or false by comparing, composing, and/or decomposing the numbers without solving.

Determine the unknown whole number in an equation relating four whole numbers using comparative relational thinking. For example, solve $76 + 9 = n + 5$ for $n$ by arguing that nine is four more than five, so the unknown number must be four greater than 76. **MAFS.4.OA.1.b**

### Students will:
- **NOTE:** Due to this standard’s emphasis on mental math, it will be embedded through the use of Number Talks.
- **compare** the two sides of an equation to determine the unknown value without solving.

**NOTE:** Equations may include:
- Addition and subtraction with 1,000.
- Multiplication of 2-digit by 1-digit.
- Division of 2-digit by 1-digit or a multiple of 10 by 1-digit.

1. Make sense of problems and persevere in solving them. **MAFS.K12.MP.1.1**
2. Reason abstractly and quantitatively. **MAFS.K12.MP.2.1**
6. Attend to precision. **MAFS.K12.MP.6.1**

### Topic Comments:

OA.1.3 and NBT.2.4 are repeated here to include all four operations and will be finalized in topic 14. Repeating these standards throughout the year provides students multiple opportunities to develop these skills—which are major areas of focus for this grade level. MD.1.2 is repeated from the previous topic, but in this topic the emphasis is on using the four operations and all problem types. This standard will be finalized in topic 12 to include decimal fractions.

Students use various diagrams and precise language to solve measurement problems and explain their strategies (MP.1, MP.6). They make connections between abstract representations and the problem situations (MP.2).
Topic 9: Solving addition and subtraction problems involving fractions and mixed numbers

In this topic students will use their understanding of adding and subtracting fractions and generating equivalent fractions to solve problems involving fractions and mixed numbers. Students rely on their previous work with whole numbers as fractions to compose and decompose whole numbers into fractional quantities. Data is used in this topic to support students' understanding of fractional quantities both less than and greater than 1.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
</table>
| Understand a fraction \( \frac{a}{b} \) with a > 1 as a sum of fractions \( \frac{1}{b} \).  
| c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.  
| d. Solve word problems involving the addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. | **MAFS.4.NF.2.3**  
| data set  
decomposition  
denominator  
equation  
equivalent fraction  
fraction greater than 1  
line plot  
mixed number  
numerator  
sum  
unit fraction  
whole |

**Students will:**

- **decompose** a mixed number into a sum of fractions equal to 1 and a fraction less than 1 to find an equivalent fraction to replace the mixed number in an addition or subtraction situation.

  E.g.,

  \[
  2 \frac{1}{8} + 3 \frac{3}{8} = \frac{17}{8}
  \]

- **find** equivalent sums or differences by **converting** fractions greater than 1 to mixed numbers by decomposing the fraction into a sum of fractions equal to 1 and fractions less than 1.

  E.g.,

  \[
  \frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}
  \]

- **add** and **subtract** mixed numbers with like denominators using equivalent fractions, and/or by using properties of operations and the relationship between addition and subtraction.

  E.g.,

  \[
  3 \frac{3}{4} + 2 \frac{1}{4}
  \]

<table>
<thead>
<tr>
<th>Student 1:</th>
<th>Student 2:</th>
<th>Student 3:</th>
</tr>
</thead>
</table>
  | 3 + 2 = 5 and \( \frac{3}{4} + \frac{1}{4} = 1 \)  
so 5 + 1 = 6 | \( 3 \frac{3}{4} + 2 \frac{1}{4} = \frac{5}{4} + \frac{1}{4} = \frac{5 + 1}{4} = 6 \)  
so \( \frac{5}{4} + \frac{1}{4} = \frac{6}{4} = 6 \) | \( 3 \frac{3}{4} = \frac{9}{4} \) and \( 2 \frac{1}{4} = \frac{9}{4} \)  
so \( \frac{9}{4} + \frac{9}{4} = \frac{18}{4} = 6 \) |
E.g., Students use the relationship between addition and subtraction to solve $3\frac{1}{4} - 1\frac{1}{4}$ by thinking of this expression as an unknown addend problem.

\[
\begin{align*}
1\frac{3}{4} + ? &= 3\frac{1}{4} \\
\frac{1}{4} + 1 + \frac{1}{4} &= \frac{2}{4} \\
3\frac{1}{4} - 3\frac{1}{4} &= 1\frac{2}{4}
\end{align*}
\]

- **solve** word problems involving addition and subtraction of fractions and mixed numbers with like denominators using visual fraction models (i.e., circular models, rectangular models, and number line models) and/or equations.

E.g., Lindsey and Brooke need $3\frac{3}{8}$ feet of ribbon to design costumes for a performance. Lindsey has $1\frac{1}{8}$ and Brooke has $2\frac{5}{8}$ feet of ribbon. If they combine what they have, will that be enough for the project? Explain why or why not.

Ribbon needed for the project:

\[
3\frac{3}{8} = \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{3}{8} = \frac{27}{8}
\]

Lindsey's $\frac{9}{8}$ ft. + Brooke's $\frac{21}{8}$ ft. = $\frac{30}{8}$ ft.

$\frac{30}{8}$ ft. is greater than $\frac{27}{8}$ ft. So, they have enough ribbon for the costumes.

**NOTE:** Denominators of given fractions are limited to: 2, 3, 4, 5, 6, 8, 10, 12, 100.
Make a line plot to display a data set of measurements in fractions of a unit (\(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\)). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

MAFS.4.MD.2.4

Students will:
- **create** a line plot to display measurement data including fraction units of halves, quarters, and eighths.
- **use** the measurement data on a line plot to solve problems involving the addition and subtraction of fractions with like denominators.

E.g., A small group of students in Mr. Tordini’s class measure the antennae of different insects to the nearest eighth of an inch. They display the measurements on the line plot shown below. Answer the following questions based on the line plot.

![Insect Antennae](image)

**Length of antennae in inches**

- What is the difference between the longest and shortest antennae of all the insects that were measured?
  (Answer: \(\frac{12}{8} - \frac{1}{8} = \frac{11}{8}\) inches)
- If the antennae with the longest length were laid end to end, what would their total length be?
  (Answer: \(\frac{12}{8} + \frac{12}{8} = \frac{24}{8}\) inches)

2. Reason abstractly and quantitatively.
4. Model with mathematics.

MAFS.K12.MP.2.1
MAFS.K12.MP.4.1

**Topic Comments:**

4.MD.2.4 extends students’ work from Grade 3 with simple fractions on a line plot (3.MD.2.4) to include eighths and to now solve addition and subtraction problems using the data.

Students reason about fractions by using abstract models to represent both the data and the fractional quantities (MP.2, MP.4).
### Topic 10: Angle Measurement

This topic is an introduction to angles and angle measurement. Students start this topic drawing points, lines, segments, rays and angles since it is foundational to the other standards in this topic. Students use their understanding of equal partitioning and unit measurement to understand angle and turn measure.

#### Standards

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.</td>
<td><strong>MAFS.4.MD.3.5</strong></td>
</tr>
</tbody>
</table>
| a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through \( \frac{1}{360} \) of a circle is called a “one-degree angle,” and can be used to measure angles. | **acute angle**  
**angle measure**  
**degree**  
**endpoint**  
**line**  
**line segment**  
**obtuse angle**  
**one-degree angle**  
**point**  
**protractor**  
**ray**  
**right angle**  
**rotation** |
| b. An angle that turns through \( n \) one-degree angles is said to have an angle measure of \( n \) degrees. | |

#### Students will:

- **recognize** an angle as a geometric shape that is formed when two rays share a common endpoint.
- **explain** the relationship between a circle and the number of degrees in an angle (i.e., an angle is measured in reference to a circle—its center is the endpoint for each of the rays that make up the angle).
- **explain** an angle as a series of “one-degree turns” and the total number of “one-degree turns” is the measure of the angle in degrees (°).

E.g., A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 one-degree turns, what is the measure of the sprinkler’s rotation in degrees?

- **explain** that since it takes 360 “one-degree turns” to rotate through a circle, \( \frac{1}{360} \) of a circle is a “one-degree angle”.

E.g.,

![Images of angles: 45°, 90°, 180°, 270°, 360°]
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. MAFS.4.MD.3.6

**Students will:**

- **measure** an angle to the nearest whole number degrees using a protractor.

  E.g.,

  ![Protractor Images]

  120 degrees 135 degrees

- **use** a protractor to sketch an angle given a specific measurement.

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. MAFS.4.G.1.1

**Students will:**

- **draw** points, lines, line segments, rays, right angles (exactly 90°), acute angles (less than 90°), obtuse angles (greater than 90° and less than 180°).

  E.g.,

  ![Geometric Shapes]

4. Model with mathematics.
5. Use appropriate tools strategically. MAFS.K12.MP.4.1 MAFS.K12.MP.5.1

**Topic Comments:**

In this topic, 4.G.1.1 focuses on drawing points, lines, line segments, rays, and different types of angles. The standard will be finalized in topic 13.

Students select and use a protractor to measure angles and represent the angles with drawings (MP.4, MP.5).
**Topic 11: Multiplying fractions by whole numbers**

In this topic students apply their understanding of composing and decomposing fractions to develop a conceptual understanding of multiplication of a fraction by a whole number. Students also use and extend their previous understandings of operations with whole numbers and relate that understanding to fractions.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret a multiplication equation as a comparison, e.g., interpret (35 = 5 \times 7) as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</td>
<td><strong>MAFS.4.OA.1.1</strong> decompose denominator equivalent fraction multiplicative comparison numerator times as many times as much unit fraction whole number</td>
</tr>
</tbody>
</table>

**Students will:**

- **interpret** an equation involving multiplying a fraction by a whole number as a verbal comparison (e.g., if \(\frac{5}{4} = 5 \times \frac{1}{4}\), then \(\frac{5}{4}\) is 5 times as much as \(\frac{1}{4}\)).
- **represent** verbal statements of multiplicative comparison involving multiplying a fraction by a whole number as a multiplication equation (e.g., Rob's stick is 6 times as long as Todd's. Todd's stick is \(\frac{1}{5}\) yard long. A multiplication equation that represents the situation is \(6 \times \frac{1}{5} = \frac{6}{5}\).)

**NOTE:** Multiplying by a fraction (i.e., \(\frac{a}{b} \times n\)) is not an expectation of the Standards in Grade 4, so situations should be limited to contexts where the multiplier is a whole number and the number being multiplied can realistically be represented by a fraction (such as measurement situations).

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

- **a.** Understand a fraction \(\frac{a}{b}\) as a multiple of \(\frac{1}{b}\). For example, use a visual fraction model to represent \(\frac{5}{4}\) as the product \(5 \times \frac{1}{4}\), recording the conclusion by the equation \(\frac{5}{4} = 5 \times \frac{1}{4}\).

- **b.** Understand a multiple of \(\frac{a}{b}\) as a multiple of \(\frac{1}{b}\), and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express \(3 \times \frac{2}{5}\) as \(6 \times \frac{1}{5}\), recognizing this product as \(\frac{6}{5}\). (In general, \(n \times \left(\frac{a}{b}\right) = \left(n \times \frac{a}{b}\right)\).)

- **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g. by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat \(\frac{3}{8}\) of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

**Students will:**

- **decompose** a fraction into unit fractions (i.e., fractions with a numerator of one) and record the conclusion with a multiplication equation.

\[
\begin{array}{c}
\frac{1}{7} \\
\frac{1}{7} \\
\frac{1}{7}
\end{array}
\]

3 groups of \(\frac{1}{7}\) is \(\frac{3}{7}\), so \(3 \times \frac{1}{7} = \frac{3}{7}\)
- **recognize** that a fraction \( \frac{a}{b} \) is a multiple of \( \frac{1}{b} 
- **recognize** that a multiple of \( \frac{2}{b} \) is also a multiple of \( \frac{1}{b} \) (e.g., 3 groups of \( \frac{2}{5} \) has the same value as 6 groups of \( \frac{1}{5} \)) and use this understanding to **multiply** a fraction by a whole number.

E.g., \( 3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5} \)

- **solve** word problems that involve multiplying a fraction by a whole number using visual models and equations.

E.g., If each person at a party eats \( \frac{3}{8} \) of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed?

\[
\begin{align*}
3/8 + 3/8 + 3/8 + 3/8 + 3/8 &= 15/8 = 1\frac{7}{8}
\end{align*}
\]

**NOTE:** Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

1. Make sense of problems and persevere in solving them. 
6. Attend to precision.

**MAFS.K12.MP.1**

**MAFS.K12.MP.6**

**Topic Comments:**

4.OA.1.1 is readdressed in this topic to include multiplication of fractions by whole numbers and apply the understanding of “times as much” (i.e. multiplication as comparison) to multiplying a fraction by a whole number.

4.NF.2.4 is limited to multiplying fractions by whole numbers (e.g., \( 3 \times \frac{2}{5} \)). Students do not multiply whole numbers by fractions (e.g., \( \frac{2}{5} \times 3 \)) or fractions by fractions until Grade 5.

Students use precise language to communicate their comprehension of the problem situations and defend their various solution methods (MP.1, MP.6)
### Unit 4

**PACING:** April 1 – May 31

**Topic 12: Comparing decimal fractions and understanding notation**

In this topic, students use their previous work with fractions to represent special fractions in a new way. Students use their understanding of equivalent fractions to begin to use decimal notation—however, it is not the intent at this grade level to connect this notation to the base-ten system. The focus is on solving word problems involving simple fractions or decimals. Work with money can support this work with decimal fractions.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
</table>
| Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. For example, express \( \frac{3}{10} \) as \( \frac{30}{100} \), and add \( \frac{3}{10} + \frac{4}{100} = \frac{34}{100} \). | MAFS.4.NF.3.5

**Students will:**

- **represent** fractions with a denominator of 10 and fractions with a denominator of 100 using models (e.g., grids, base ten blocks, money, and number lines).
- **express** a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.

E.g.,

![Tenths Grid](image1)

![Hundredths Grid](image2)

\( \frac{3}{10} = \frac{30}{100} \)

**NOTE:** While students may *discover* the pattern of additional zeros at the end of numerators and denominators, they need to be able to justify this pattern in terms of fraction equivalence.

- **add** two fractions with respective denominators 10 and 100 by finding equivalent fractions with like denominators.
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe length as 0.62 meters; locate 0.62 on a number line diagram.

<table>
<thead>
<tr>
<th>Students will:</th>
</tr>
</thead>
</table>
| • demonstrate place value of decimals through the hundredths using concrete materials (e.g., decimal grids, base ten blocks, number lines).
|   E.g.,
|     | Hundreds | Tens | Ones | • | Tenths | Hundredths |
| • translate a fraction with a denominator of 10 or 100 into its equivalent decimal form.
| • translate a decimal to the tenths or hundredths place into its equivalent fraction form.
| • represent a decimal value on a number line.

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

<table>
<thead>
<tr>
<th>Students will:</th>
</tr>
</thead>
</table>
| • explain that decimals can only be compared when they refer to the same size wholes.
| NOTE: Decimals may be greater than 1.
| • compare decimals with and without models (such as decimal grids, money, or base ten blocks) and record the comparison numerically using symbols: $<$, $>$ or $=$.
| • justify the comparison by reasoning about the size of the decimal.
| E.g., To compare 1.32 and 1.6, students create a model to show that $1.32 < 1.6$
Use the four operations to solve word problems involving distances, intervals of time, and money, including problems involving simple fractions or decimals. Represent fractional quantities of distance and intervals of time using linear models.

**MAFS.4.MD.1.2**

**NOTE:** This is an amended Florida Standard.

**Students will:**

- **use** the four operations to solve word problems involving distances (i.e., inch, feet, yard, mile; millimeter, centimeter, meter, kilometer), including simple fractions or decimals and **represent** fractional quantities of distance using linear models.

  E.g., Billy has been training for a half-marathon. Each day he runs on the treadmill $2\frac{3}{4}$ miles and runs on the outdoor track for $3\frac{1}{4}$ miles. In all, how many miles does Billy run each day?

  **NOTE:** Items could involve conversions from larger units to smaller units. Students need to be provided with the Grade 4 FSA Mathematics Reference Sheet in order to practice using the tool.

  E.g., Keisha and Juan were training for a race. Keisha ran 3 kilometers twice a week and 1 kilometer once a week. Juan ran 2,500 meters three times a week. Who ran the farthest during the week?

- **use** the four operations to solve word problems involving intervals of time including simple fractions and **represent** these intervals of time using linear models.

  E.g., Jim got to the fair at 5:45 P.M. He spent $1\frac{1}{4}$ hours playing games, $1\frac{1}{4}$ hours riding rides, and $\frac{1}{4}$ hour eating dinner before leaving. What time did Jim leave?

- **use** the four operations to solve word problems involving money including simple decimals.

  E.g., Helen bought popcorn at the movies. She bought 2 large bags, and each one cost $13.00. How much change did she receive from $50.00?

**MAFS.K12.MP.3.1** **MAFS.K12.MP.7.1**

**3. Construct viable arguments and critique the reasoning of others.**

**7. Look for and make use of structure.**

**Topic Comments:**

**4.MD.1.2** was addressed in topics 7 & 8 using whole numbers. It is important to note that students are not expected to do computations with quantities in decimal notation. Students can use visual fraction models to solve problems involving simple fractions or decimals.

Students compare decimals fractions and justify their comparisons using either a fraction model or their understanding of the notation (MP.3, MP.7).
## Topic 13: Recognizing and analyzing attributes of 2-dimensional shapes

**Pacing:** April 15 - 30

In this topic students develop their spatial reasoning skills by using a wide variety of attributes to talk about 2-dimensional shapes. Students analyze geometric figures based on angle measurement, parallel and perpendicular lines, and symmetry.

### Standards

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</td>
<td>MAFS.4.OA.3.5 attributes acute angle additive angle angle measure category classify decompose degree endpoint feature generate line line segment line symmetry numeric pattern obtuse angle parallel lines perpendicular lines point ray right angle right triangle rule sequence shape pattern term variable</td>
</tr>
<tr>
<td><strong>Students will:</strong></td>
<td></td>
</tr>
<tr>
<td>• <strong>generate</strong> a number or shape pattern that follows a given one- or two-step rule.</td>
<td></td>
</tr>
<tr>
<td>• <strong>identify</strong> features in number or shape patterns after following a given one- or two-step rule.</td>
<td></td>
</tr>
<tr>
<td>E.g., The pattern repeats every five steps and goes in this order, circle, hexagon, right triangle, square, square. Can you draw the next four shapes in the pattern?</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Pattern Images" /></td>
<td></td>
</tr>
<tr>
<td>Can you tell what the 51st shape in the pattern will be without drawing all of the shapes? How did you determine what the 51st shape would be?</td>
<td></td>
</tr>
<tr>
<td>“The pattern is circle, hexagon, right triangle, square, square, and the pattern repeats every five numbers. I looked at 50 and knew the 50th shape would be a square, so the 51st shape will begin the pattern again and it is a circle.”</td>
<td></td>
</tr>
<tr>
<td>Recognize angle measure as additive. When an angle is decomposed into nonoverlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</td>
<td>MAFS.4.MD.3.7</td>
</tr>
<tr>
<td><strong>Students will:</strong></td>
<td></td>
</tr>
<tr>
<td>• <strong>recognize</strong> angle measure as additive, explaining that the angle measurement of a larger angle is the sum of the angle measures of its decomposed parts.</td>
<td></td>
</tr>
<tr>
<td>E.g.,</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Angle Diagram" /></td>
<td></td>
</tr>
<tr>
<td>35 + 90 = b</td>
<td></td>
</tr>
<tr>
<td>• <strong>solve</strong> addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.</td>
<td></td>
</tr>
<tr>
<td>E.g., For this right angle, what is the value of x?</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Right Angle Diagram" /></td>
<td></td>
</tr>
<tr>
<td>60 + x = 90</td>
<td></td>
</tr>
</tbody>
</table>
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

**Students will:**
- **draw** points, lines, line segments, rays, right angles (exactly 90°), acute angles (less than 90°), obtuse angles (greater than 90° and less than 180°), and perpendicular and parallel lines.
  
  E.g.,

- **identify** points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional shapes.

  E.g., List the two-dimensional attributes within this trapezoid:

  ![Trapezoid with labels](image)

  **Student:** two acute angles, four line segments, a pair of parallel lines, etc.

**NOTE:** Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as greater than (obtuse) or less than (acute) a right angle.

Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

**Students will:**
- **classify** two-dimensional shapes into the following categories: those with parallel lines, those with perpendicular lines, those with both parallel and perpendicular lines, those with no parallel or perpendicular lines.
- **classify** two-dimensional shapes into categories based on the presence or absence of acute, obtuse, or right angles.
- **recognize** that right triangles form a category of triangles and **identify** triangles that fit in this category.

  E.g., Do you agree with the label on each of the ovals in the Venn diagram? Why or why not? Explain why some shapes fall in the overlapping sections of the ovals.
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

MAFS.4.G.1.3

Students will:

- explain line symmetry and identify figures that have line symmetry (e.g., fold a figure or draw a line so it has two parts that match exactly).
- draw lines of symmetry in both regular and non-regular polygons.

E.g.,

![Line Symmetry Examples](image)

**NOTE:** This standard only includes line symmetry, NOT rotational symmetry.

3. Construct viable arguments and critique the reasoning of others.
4. Use appropriate tools strategically.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

MAFS.K12.MP.3.1  
MAFS.K12.MP.5.1  
MAFS.K12.MP.7.1

**Topic Comments:**

In this topic, **4.OA.3.5** includes repeated and growing shape patterns. **4.G.1.1** was first addressed in topic 10, and is finalized in this topic to include perpendicular and parallel lines and to include identifying points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines in two-dimensional shapes.

The concepts in this topic lend themselves to using technology applications (**MP.5**). Students understand that geometric figures can be classified by analyzing various properties (**MP.7**) and justify their conclusions by using viable arguments (**MP.3**).
### Topic 14: Revisiting problem solving with whole numbers

This is a culminating topic in which students focus on problem solving in order to demonstrate fluency with the standard algorithms in addition and subtraction. They demonstrate computational fluency with all addition and subtraction problem types. All standards in this topic have been addressed in prior topics. These concepts require greater emphasis due to the depth of the ideas, the time they take to master, and/or their importance to future mathematics.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
</table>
| Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. | MAFS.4.OA.1.2  
Students will:  
- **distinguish** multiplicative comparison (e.g., Tonya has 3 times as many cousins as Matthew) from additive comparison (e.g., Tonya has 3 more cousins than Matthew).  
- **solve** word problems involving multiplicative comparison (situations may include 2-digit by 1-digit). | area model  
multiplicative comparison  
properties of operations  
rectangular array  
standard algorithm |
| Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. | MAFS.4.OA.1.3  
Students will:  
- **solve** multi-step word problems (i.e., up to 3 steps) that involve the four operations using strategies for this grade level (e.g., rectangular arrays, area models, properties of operations, standard algorithms for addition and subtraction, etc.).  
- **represent** a multi-step word problem using equations involving a variable represented by a letter for the unknown number.  
- **interpret** remainders that result from multi-step word problems.  
- **assess** the reasonableness of answers to multi-step word problems using estimation strategies and mental computation prior to calculating.  
NOTE: The expectations for multi-step word problems include addition and subtraction within 1,000, multiplication of 2-digit by 1-digit, and division of 2-digit by 1-digit. | |
| Fluently add and subtract multi-digit whole numbers using the standard algorithm. | MAFS.4.NBT.2.4  
Students will:  
- **fluently** add and subtract multi-digit whole numbers using the standard algorithms.  
NOTE: Addition expressions may contain up to three addends. | |
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**MAFS.4.NBT.2.5**

**Students will:**
- **apply** an understanding of rectangular arrays and area models to connect to the partial products strategy.
- **solve** multi-digit multiplication problems that extend to 4-digit by 1-digit or up to 2-digit by 2-digit using strategies based on place value and the properties of operations and **explain** the calculation by using equations.

E.g.,

\[
\begin{array}{c}
23 \\
\times\quad 18 \\
\hline
24 \\
160 \\
30 \\
200 \\
\hline
414
\end{array}
\]

Partial Products Strategy

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**MAFS.4.NBT.2.6**

**Students will:**
- **apply** an understanding of rectangular arrays and area models to connect to the partial quotients strategy.
- **solve** division of a multi-digit number (with up to four digits) by a one-digit number using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division, and **explain** the calculation using equations.

E.g.,

Partial Quotients Strategy

Multiplying Up strategy
| 2. Reason abstractly and quantitatively. | MAFS.K12.MP.2.1 |
| 8. Look for and express regularity in repeated reasoning. | MAFS.K12.MP.8.1 |

**Topic Comments:**

NBT.2.5 & NBT.2.6 are finalized in this topic to include equations and strategies such as partial products and partial quotients. The use of these strategies is a step toward the standard algorithms for multiplication and division. Students are still building conceptual understanding of these two operations in Grade 4 and the standard algorithms are not used until Grades 5 and 6. This being said, these strategies were reserved for later in the scope and sequence as a bridge to the algorithms, after conceptual understanding has been addressed throughout the year through the use of arrays and area models.

In demonstrating fluency, students explain and flexibly use properties of operations and place value to solve problems, looking for shortcuts and applying generalized strategies (MP.2, MP.8).
Critical Areas for Mathematics in Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of division to find quotients involving multi-digit dividends; (2) developing understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (e.g., equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.
# Grade 4 Major, Supporting, and Additional Work

<table>
<thead>
<tr>
<th>Topic</th>
<th>Title</th>
<th>Major Work</th>
<th>Supporting Work</th>
<th>Additional Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Applying place value concepts in whole number addition and subtraction</td>
<td>4.NBT.1.1, 4.NBT.1.2, 4.NBT.1.3, 4.NBT.2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Exploring multiples and factors</td>
<td></td>
<td>4.OA.2.4</td>
<td>4.OA.3.5</td>
</tr>
<tr>
<td>3</td>
<td>Using multiplication and division strategies with larger numbers</td>
<td>4.OA.1.3, 4.NBT.2.5, 4.NBT.2.6, 4.OA.1.a, 4.OA.1.b</td>
<td></td>
<td>4.MD.1.3</td>
</tr>
<tr>
<td>4</td>
<td>Decomposing and composing fractions for addition and subtraction</td>
<td>4.NF.2.3 (a,b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Understanding fraction equivalence and comparison</td>
<td>4.NF.1.1, 4.NF.1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Introducing measurement conversions</td>
<td>4.OA.1.1, 4.NBT.1.1</td>
<td>4.MD.1.1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Solving problems using multiplicative comparison</td>
<td>4.OA.1.2, 4.NBT.1.3, 4.OA.1.a, 4.OA.1.b</td>
<td></td>
<td>4.MD.1.2</td>
</tr>
<tr>
<td>8</td>
<td>Solving measurement problems using the four operations</td>
<td>4.OA.1.3, 4.NBT.2.4, 4.OA.1.a, 4.OA.1.b</td>
<td></td>
<td>4.MD.1.2</td>
</tr>
<tr>
<td>9</td>
<td>Solving addition and subtraction problems involving fractions and mixed numbers</td>
<td>4.NF.2.3 (c,d)</td>
<td></td>
<td>4.MD.2.4</td>
</tr>
<tr>
<td>10</td>
<td>Angle measurement</td>
<td>4.G.1.1</td>
<td></td>
<td>4.MD.3.5 (a,b), 4.MD.3.6</td>
</tr>
<tr>
<td>11</td>
<td>Multiplying fractions by whole numbers</td>
<td>4.OA.1.1, 4.NF.2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Comparing decimal fractions and understanding notation</td>
<td>4.NF.3.5, 4.NF.3.6, 4.NF.3.7</td>
<td></td>
<td>4.MD.1.2</td>
</tr>
<tr>
<td>14</td>
<td>Problem solving with whole numbers</td>
<td>4.OA.1.2, 4.OA.1.3, 4.NBT.2.4, 4.NBT.2.5, 4.NBT.2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Revisiting solving addition and subtraction problems involving fractions and mixed number data present in line plots</td>
<td>4.NF.2.3 (c,d)</td>
<td></td>
<td>4.MD.2.4</td>
</tr>
<tr>
<td>16</td>
<td>Revisiting using conversions to solve measurement problems</td>
<td></td>
<td></td>
<td>4.MD.1.1, 4.MD.1.2</td>
</tr>
</tbody>
</table>
Standards for Mathematical Practice

Grade 4 students will:

1. **Make sense of problems and persevere in solving them.** (SMP.1)
   Mathematically proficient students in Grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2. **Reason abstractly and quantitatively.** (SMP.2)
   Mathematically proficient students in Grade 4 recognize that a number represents a specific quantity. They extend this understanding from whole numbers to their work with fractions and decimals. This involves two processes- decontextualizing and contextualizing. Grade 4 students decontextualize by taking a real-world problem and writing and solving equations based on the word problem. For example, consider the task, “If each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Students will decontextualize by writing the equation 3/8 × 5 or repeatedly add 3/8 5 times. While students are working they will contextualize their work - knowing that the answer 15/8 or 1 7/8 represents the total number of pounds of roast beef that will be needed. Further, Grade 4 students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

3. **Construct viable arguments and critique the reasoning of others.** (SMP.3)
   Mathematically proficient students in Grade 4 construct arguments using concrete representations, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. Students refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking through discussions and written responses.

4. **Model with mathematics.** (SMP.4)
   Mathematically proficient students in Grade 4 represent problem situations in various ways, including writing an equation to describe the problem. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Grade 4 students should evaluate their results in the context of the situation and reflect on whether the results make sense.

5. **Use appropriate tools strategically.** (SMP.5)
   Mathematically proficient students in Grade 4 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

6. **Attend to precision.** (SMP.6)
   Mathematically proficient students in Grade 4 develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

7. **Look for and make use of structure.** (SMP.7)
   Mathematically proficient students in Grade 4 closely examine numbers to discover a pattern or structure. For instance, students use properties of operations to explain calculations (area model). They relate representations of counting problems such as arrays to the multiplication principal of counting. They generate number and shape patterns that follow a given rule.

8. **Look for and express regularity in repeated reasoning.** (SMP.8)
   Mathematically proficient students in Grade 4 notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.
<table>
<thead>
<tr>
<th>Common Addition and Subtraction Situations Table</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Result Unknown</strong></td>
</tr>
<tr>
<td><strong>Add to</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Take from</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Total Unknown</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Put Together/Take Apart</strong></td>
</tr>
<tr>
<td><strong>Difference Unknown</strong></td>
</tr>
<tr>
<td><strong>Compare</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity, National Research Council, 2009, pp. 32–33.

1 This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean “makes” or “results in” but always means “is the same number as.” Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all the decompositions of a number and reflecting on the patterns involved.

2 Either addend can be unknown; both variations should be included.
## Common Multiplication and Division Situations Table

<table>
<thead>
<tr>
<th>Unknown Product</th>
<th>Group Size Unknown (“How many in each group?” Division)</th>
<th>Number of Groups Unknown (“How many groups?” Division)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 6 = ?$</td>
<td>$3 \times ? = 18$ and $18 \div 3 = ?$</td>
<td>$? \times 6 = 18$ and $18 \div 6 = ?$</td>
</tr>
<tr>
<td><strong>Equal Groups</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>There are 3 bags with 6 plums in each bag. How many plums are there in all?</td>
<td>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</td>
<td>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</td>
</tr>
<tr>
<td><strong>Arrays</strong>, <strong>Area</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>There are 3 rows of apples with 6 apples in each row. How many apples are there?</td>
<td>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</td>
<td>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</td>
</tr>
<tr>
<td>Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</td>
<td>Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</td>
<td>Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</td>
</tr>
<tr>
<td><strong>Larger Unknown</strong></td>
<td><strong>Smaller Unknown</strong></td>
<td><strong>Multiplier Unknown</strong></td>
</tr>
<tr>
<td>A blue hat costs $6. A red hat cost 3 times as much as the blue hat. How much does the red hat cost?</td>
<td>A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does the blue hat cost?</td>
<td>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?</td>
</tr>
<tr>
<td>Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</td>
<td>Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</td>
<td>Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</td>
</tr>
<tr>
<td><strong>Smaller Unknown</strong></td>
<td><strong>Larger Unknown</strong></td>
<td><strong>Multiplier Unknown</strong></td>
</tr>
<tr>
<td>$\frac{1}{3} \times 18 = ?$</td>
<td>$\frac{1}{3} \times ? = 6$</td>
<td>$? \times 18 = 6$</td>
</tr>
<tr>
<td>A red hat costs $18. A blue hat costs $6 as much as the red hat. How much does the blue hat cost?</td>
<td>A blue hat costs $6 and that is $\frac{1}{3}$ of the cost of a red hat. How much does a red hat cost?</td>
<td>A red hat costs $18 and a blue hat costs $6. What fraction of the cost of the red hat is the cost of the blue hat?</td>
</tr>
<tr>
<td>General</td>
<td>General</td>
<td>General</td>
</tr>
<tr>
<td>$a \times b = ?$</td>
<td>$a \times ? = p$ and $p \div a = ?$</td>
<td>$? \times b = p$ and $p \div b = ?$</td>
</tr>
</tbody>
</table>

1. The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
2. The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are there? Both forms are valuable.
3. Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
4. Multiplicative Compare problems appear first in Grade 4, with whole-number values for $A$, $B$, and $C$, and with the “times as much” language in the table. In Grade 5, unit fractions language such as “one third as much” may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., “A red hat costs $A$ times as much as the blue hat” results in the same comparison as “A blue hat costs $\frac{1}{3}$ times as much as the red hat,” but has a different subject.
Grade 4 FSA Mathematics Reference Sheet

Customary Conversions

1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet
1 mile = 1,760 yards

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 pound = 16 ounces
1 ton = 2,000 pounds

Metric Conversions

1 meter = 100 centimeters
1 meter = 1000 millimeters
1 kilometer = 1000 meters

1 liter = 1000 milliliters

1 gram = 1000 milligrams
1 kilogram = 1000 grams

Time Conversions

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 days
1 year = 52 weeks

Formulas

\[ A = lw \]

\[ P = 2l + 2w \]