# Elementary Instructional Math Block

<table>
<thead>
<tr>
<th>Time</th>
<th>Components</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-15 minutes</td>
<td>Number Talks</td>
<td>Short, daily fluency routine that engages students in meaningful conversations around purposefully crafted computation problems that are solved using number relationships and the structure of numbers. Students are asked to communicate their thinking when presenting and justifying solutions to problems they solve mentally while the teacher records their ideas with mathematical precision. These exchanges lead to the development of more accurate, efficient, and flexible strategies.</td>
</tr>
<tr>
<td>5 minutes</td>
<td>Opening: Hook/Coherence Connection</td>
<td>The teacher will engage students to create interest for the whole group lesson or review prerequisite standards to prepare students to make explicit connections that will allow students to apply and extend previous learning when interacting with the lesson’s grade-level content.</td>
</tr>
<tr>
<td>15 minutes</td>
<td>Whole Group: Mini Lesson/Guided Practice</td>
<td>Used prior to small group to introduce/practice new knowledge and skills or after small group to refine/practice strategies discovered by students. The lesson focuses on the depth of grade-level cluster(s), grade-level content standard(s), or part(s) thereof, intentionally targeting the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed. During this time, the teacher makes the mathematics of the lesson explicit using clear and correct explanations, representations, tasks, and/or examples. The teacher provides opportunities for all students to work with and practice grade-level problems and exercises, deliberately checking for understanding throughout the lesson and adapting the lesson according to student understanding. The teacher poses high-quality questions and problems that prompt students to share their developing thinking about the content of the lesson. Class created anchor charts are constructed by strategically adding key concepts throughout the topic’s lessons.</td>
</tr>
<tr>
<td>30-40 minutes</td>
<td>Small Collaborative Groups/Independent Practice</td>
<td>The teacher encourages reasoning and problem solving by posing challenging problems that offer opportunities for student choice of appropriate tools and promote productive struggle. Students work in small, flexible collaborative groups to engage in mathematical tasks while the teacher circulates and asks questions to elicit thinking, providing support or extensions as needed. The teacher asks students to explain and justify work, connecting and developing students’ informal language to precise mathematical language appropriate to their grade, and provides feedback that helps students revise initial work. The teacher makes observations to select and sequence appropriate strategies for students to share during the class discussion.</td>
</tr>
<tr>
<td>5 minutes</td>
<td>Closure: Summarize</td>
<td>The teacher strengthens all students’ understanding of the content by strategically sharing a variety of students’ representations and solution methods. The teacher facilitates the summary of the mathematics with references to student work and by creating the conditions for student conversations where students are encouraged to talk about each other’s thinking in order to reinforce the purpose of the lesson.</td>
</tr>
</tbody>
</table>

Formative techniques occur throughout the framework to drive instruction, guide collaborative grouping, and evaluate which students will need intervention/enrichment.
# Grade 5 Math Instructional Calendar

<table>
<thead>
<tr>
<th>Units</th>
<th>Topics</th>
<th>Standards</th>
<th>Suggested Dates</th>
</tr>
</thead>
</table>
| 1     | Understanding volume (10 days) | 5.MD.3.3  
5.MD.3.4 | Aug. 13-24 |
| 2     | Expanding understanding of place value of decimals (10 days) | 5.NBT.1.1  
5.NBT.1.2  
5.NBT.1.3 (a) | Aug. 27 - Sept. 10  
Sept. 3 (Labor Day) |
| 3     | Developing multiplication and division strategies (15 days) | 5.NBT.2.5 (not fluently)  
5.NBT.2.6 | Sept. 11 - Oct. 2  
Sept. 17 (TDD) |
| 4     | Using equivalency to add and subtract fractions with unlike denominators (12 days) | 5.NF.1.1  
5.NF.1.2 | Oct. 3-19  
Oct. 15 (TDD) |
| 5     | Understanding the concept of multiplying fractions by fractions (13 days) | 5.NF.2.3  
5.NF.2.4 | Oct. 22- Nov. 7 |
| 6     | Comparing and rounding decimals (10 days) | 5.NBT.1.3  
5.NBT.1.4 | Nov. 8-29  
Nov. 12 (Veterans Day)  
Nov. 19-23 (Thanksgiving) |
| 7     | Interpreting multiplying fractions as scaling (12 days + 2 Flex Days) | 5.NF.2.5  
5.NF.2.6 | Nov. 30 - Dec. 19  
Dec. 20 (TDD)  
Dec. 21 - Jan. 6 (Winter Break) |
| 8     | Developing the concept of dividing unit fractions (10 days) | 5.NF.2.7 | Jan. 7-18  
Jan. 21 (MLK) |
| 9     | Solving problems involving volume (10 days) | 5.MD.3.5 | Jan. 22- Feb. 4 |
| 10    | Performing operations with decimals (14 days) | 5.NBT.2.7  
5.MD.1.1 | Feb. 5-25  
Feb. 18 (President’s Day) |
| 11    | Classifying two-dimensional geometric figures (10 days) | 5.G.2.3  
5.G.2.4 | Feb. 26- Mar. 11 |
| 12    | Solving problems with fractional quantities (12 days) | 5.MD.2.2  
5.NF.1.2  
5.NF.2.6  
5.NF.2.7 (c-only whole numbers by unit fractions) | Mar. 12- April 4  
March 15 (TDD)  
March 18-22 (Spring Break) |
| 13    | Representing algebraic thinking (8 days) | 5.OA.1.1  
5.OA.1.2 | Apr. 5-16 |
| 14    | Exploring the coordinate plane (10 days) | 5.OA.2.3  
5.G.1.1  
5.G.1.2 | Apr. 17-30 |
| 15    | Revisiting multiplication and division with whole numbers (12 days) | 5.NBT.2.5  
5.NBT.2.6 | May 1-16 |
| 16    | Revisiting problem solving with fractions (10 days) | 5.NF.1.2  
5.NF.2.6  
5.NF.2.7 (c) | May 17-31  
May 27 (Memorial Day) |
# Unit 1

**PACING:** August 13 – October 19

## Topic 1: Understanding Volume

Students expand their understanding of geometric measurement and spatial structuring to include volume as an attribute of three-dimensional space. In this topic, students develop this understanding using concrete models to discover strategies for finding volume, whereas in topic 9 students generalize this understanding in real-world problems and apply strategies and formulas. Volume is addressed in two topics (topic 1 and topic 9) because it is a major emphasis in Grade 5. The connection to multiplication and addition provides an opportunity for students to start the year off by applying the multiplication and addition strategies they learned in previous grades in a new, interesting context.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
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<tbody>
<tr>
<td>Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</td>
<td>MAFS.5.MD.3.3</td>
</tr>
<tr>
<td>a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.</td>
<td>attribute cubic units</td>
</tr>
<tr>
<td>b. A solid figure which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.</td>
<td>rectangular prism</td>
</tr>
<tr>
<td>Students will:</td>
<td>unit cube</td>
</tr>
<tr>
<td>• identify volume as an attribute of a solid figure.</td>
<td>volume</td>
</tr>
<tr>
<td>• explain that a cube with 1 unit side lengths is “one cubic unit” of volume, and understand that this cubic unit is used to measure volume.</td>
<td></td>
</tr>
<tr>
<td>• explain that the volume of a solid figure can be found by filling it with unit cubes, without gaps and overlaps, and finding the total number of unit cubes.</td>
<td></td>
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</table>

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.  

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<tr>
<td>Students will:</td>
<td>MAFS.5.MD.3.4</td>
</tr>
<tr>
<td>• measure the volume of a three-dimensional figure (i.e., rectangular prism and cube) by filling it with unit cubes and counting the number of unit cubes.</td>
<td>attribute cubic units</td>
</tr>
<tr>
<td>E.g.,</td>
<td>rectangular prism</td>
</tr>
<tr>
<td>How many cubic in. in one layer?</td>
<td>unit cube</td>
</tr>
<tr>
<td>How many total layers?</td>
<td>volume</td>
</tr>
</tbody>
</table>

**NOTE:** Items may contain right rectangular prisms with whole-number side lengths. Figures may only be shown with unit cubes. Work with exponential notation will be introduced in topic 2, therefore all measurements in this topic should be labeled as “cubic units” (i.e., cubic cm, cubic in., cubic ft.). Units$^3$ will not be used until topic 9.
<table>
<thead>
<tr>
<th>Topic Comments:</th>
<th></th>
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<tbody>
<tr>
<td>Students decompose and recompose geometric figures to make sense of the spatial structure of volume (MP.7). In particular, students explain their thinking and analyze others’ reasoning as they practice partitioning figures into layers and each layer into rows and each row into cubes (MP.3).</td>
<td></td>
</tr>
</tbody>
</table>
### Topic 2: Expanding understanding of place value of decimals

In this topic students expand their previous understanding of place value to include decimal numbers. Grade 5 is the last grade in which the NBT domain appears in the MAFS. Later work in the base-ten system relies on the meanings and properties of operations. This also contributes to deepen students’ understanding of computation and algorithms in the new domains that start in Grade 6.

<table>
<thead>
<tr>
<th>Standards</th>
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<tbody>
<tr>
<td>Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.</td>
<td>MAFS.5.NBT.1.1 base-ten numeral decimal digit expanded form exponent hundredths multi-digit number names power of 10 tenths thousandths value</td>
</tr>
</tbody>
</table>

**Students will:**
- **demonstrate** with models that in a multi-digit number, a digit in one place represents ten times the value of the place to its right.
- **demonstrate** with models that a digit in one place represents $\frac{1}{10}$ the value of the place to its left.

E.g.,

\[
\begin{array}{ccc}
5 & 5 & 5 \\
\hline
\text{grid} & \text{grid} & \text{grid} \\
\text{5 grid} & \text{5 grid} & \text{5 grid} \\
\text{5} & \text{10} & \text{100} \\
\text{or} & \text{or} & \\
\text{0.5} & \text{0.05} & \\
\end{array}
\]

- **explain** the relationship between the values of digits across multiple place values, using multiplicative comparison.

E.g., $\frac{1}{10} \times 6.4 = 0.64$; the 6 in 0.64 represents 6 tenths which is $\frac{1}{10}$ as much as the 6 ones in 6.4.

**NOTE:** While students may *discover* the pattern of “moving the decimal point”, they need to be able to justify this pattern in terms of place value. When you multiply a number by 10, the value becomes 10 times as great, so every digit shifts to the left one greater place value making it appear that the decimal point has moved to the right. E.g., $10 \times 0.04$, ten groups of 4 hundredths is 40 hundredths, which is equivalent to 4 tenths, or 0.4.

Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

<table>
<thead>
<tr>
<th>Students will:</th>
<th>MAFS.5.NBT.1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>express powers of 10 using whole-number exponents.</td>
<td>base-ten numeral decimal digit expanded form exponent hundredths multi-digit number names power of 10 tenths thousandths value</td>
</tr>
</tbody>
</table>

**NOTE:** While students may *discover* a pattern of “adding zeros”, they need to be able to justify this pattern in terms of the number of times 10 is used as a factor (as denoted by the exponent). E.g., $10^1=10$, $10^2=10 \times 10=100$, $10^3=10 \times 10 \times 10=1,000$. 
• **explain** the pattern for how and why the number of zeros in a product (when multiplying a whole number by a power of 10) relates to the power of 10.
  
  E.g., $5 \times 10^2 = 500$ because multiplying by $10^2$ is the same as using 10 as a factor two times ($5 \times 10 \times 10$) and every time a number is multiplied by 10 the digits shift one place to the left in the product, so multiplying by 10 two times shifts every digit 2 places to the left.

• **explain** the pattern in the placement of the decimal point when a decimal is multiplied by a power of 10 and how the placement of the decimal point relates to the power of 10.
  
  E.g., $1.895 \times 10^3 = 1895$ because multiplying by $10^3$ is the same as using 10 as a factor three times ($1.895 \times 10 \times 10 \times 10$) and every time a number is multiplied by 10 the digits shift one place to the left in the product, so multiplying by 10 three times shifts every digit 3 places to the left, resulting in the decimal point in the product being 3 places to the right of where it is in the factor.

• **explain** the pattern in the placement of the decimal point when a decimal is divided by a power of 10 and how the placement of the decimal point relates to the power of 10.
  
  E.g., Every time a number is divided by 10, the digits shift one place to the right in the quotient, so when dividing by 10 two times such as in the expression $15.3 \div 10^2$, every digit shifts 2 places to the right, resulting in the decimal point in the quotient being 2 places to the left of where it is in the dividend.

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**Read, write, and compare decimals to thousandths.**

<table>
<thead>
<tr>
<th>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MAFS.5.NBT.1.3</strong></td>
</tr>
</tbody>
</table>

**Students will:**

- **read and write** decimals up to the thousandths place using base-ten numerals, number names and expanded form.
  
  E.g., Some equivalent forms of $2.34$ are:

  2\begin{align*}
  &\quad + 0.30 + 0.04 \\
  &\quad 2 \times (1) + 3 \times (0.1) + 4 \times (0.01) \\
  &\quad 2 \times (1) + 3 \times (\frac{1}{10}) + 4 \times (\frac{1}{100}) \\
  &\quad 2 \times (1) + 34 \times (0.01) \\
  &\quad 2 \times (1) + 34 \times (\frac{1}{100})
  \end{align*}

  6. Attend to precision.  
  7. Look for and make use of structure.  
  **MAFS.K12.MP.6.1**  
  **MAFS.K12.MP.7.1**

**Topic Comments:**

Powers of 10 is a fundamental aspect of the base-ten system, thus **5.NBT.1.2** can help students extend their understanding of place value from millions to incorporate decimals to thousandths.

**5.NBT.1.3a** Students will be reading and writing decimals in this topic. Comparing decimals **5.NBT.1.3b** will be addressed in topic 6.

Students use their understanding of structure of whole numbers to generalize this understanding to decimals (MP.7) and explain the relationship between the numerals (MP.6).
**Topic 3: Developing multiplication and division strategies**

In this topic students build on their work from previous grade levels to refine their strategies for multiplication and division in order to reach fluency in multiplication by the end of the year. Students continue to develop more sophisticated strategies for division to become flexible and efficient with the standard algorithm in Grade 6. Students begin to find quotients with two-digit divisors early in the year to build strategies for accurate computations.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
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</thead>
<tbody>
<tr>
<td>Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
<td>MAFS.5.NBT.2.5</td>
</tr>
</tbody>
</table>

**Students will:**

- **apply** an understanding of multiplication, place value, and flexibility with multiple strategies to use the standard algorithm for multiplication.

  **NOTE:** Multiplication should not exceed 5 digits by 2 digits.

  Computational fluency is defined as accuracy, efficiency, and flexibility. Grade 5 is the first grade level in which students are expected to be fluent with the standard algorithm for multiplication.

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- **solve** division of a multi-digit dividend by a two-digit divisor using strategies based on place value, properties of operations, and/or the relationship between multiplication and division.
- **illustrate** and **explain** the calculation by using equations, rectangular arrays, and/or area models.

**NOTE:** Use of the standard algorithm for division is a Grade 6 standard.
1. Make sense of problems and persevere in solving them.

8. Look for and express regularity in repeated reasoning.

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<tbody>
<tr>
<td>In this topic 5.NBT.2.5 and 5.NBT.2.6 will focus on operations with whole numbers only. Operations with decimals will be introduced in topic 10. These standards will be finalized in topic 15, but should be practiced throughout the year to provide opportunities for students to develop proficiency with these operations.</td>
</tr>
</tbody>
</table>

Students look for regularity in their work with multiplication and division, use their understanding of the structure (MP.8) to make sense of their solutions and understand the approaches of other students (MP.1).
**Topic 4: Using equivalency to add and subtract fractions with unlike denominators**

In this topic students use what they’ve learned in Grades 3 and 4 (3.NF.1.3 & 4.NF.1.1) about equivalency in terms of visual models and benchmarks to extend understanding of adding and subtracting fractions, including mixed numbers. They reason about size of fractions to make sense of their answers—their algorithm for adding fractions. There is no mathematical reason for students to write fractions in simplest form.

### Standards

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* \( \frac{1}{2} + \frac{5}{4} = \frac{6}{12} + \frac{15}{12} = \frac{21}{12} \).  *(In general, \( \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd} \)).  

### Academic Language

**MAFS.5.NF.1.1**

denominator
equivalent fraction greater than 1mixed number numerator

**Students will:**

- **represent** addition and subtraction of fractions, including mixed numbers and fractions greater than 1, with unlike denominators using concrete models and representations. (denominators are limited to 1-20).
- **apply** concepts of equivalent fractions, and decomposition of fractions to find like denominators to add and/or subtract.

E.g.,

\[
\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}
\]

**NOTE:** Students need to be able to recognize equivalent fractions, but will **NOT** use the language, “simplify, reduce or find lowest terms”.

### Standards

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* \( \frac{2}{5} + \frac{1}{2} = \frac{3}{7} \), by observing that \( \frac{3}{7} < \frac{1}{2} \).  

### Academic Language

**MAFS.5.NF.1.2**

denominator

**Students will:**

- **solve** word problems involving addition and subtraction of fractions with like and unlike denominators.
- **use** benchmark fractions and number sense of fractions to estimate and assess reasonableness of answers.

E.g., At Lynnette’s birthday party, the boys ate \( \frac{3}{5} \) of a party sub and the girls at the party ate \( \frac{9}{12} \) of the same-size sub. Without actually adding these fractions, can you tell if more or less than a whole party sub was eaten?

The student is able to correctly explain that more than one sub was eaten because both fractions are greater than \( \frac{1}{2} \), so, their sum must be more than a whole.
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<tbody>
<tr>
<td>In this topic, <strong>5.NF.1.1</strong> involves students using the same method from Grade 4 to generate equivalent fractions (4.NF.1.1). In topic 7 students will extend this understanding of equivalency to understand that multiplying by a fraction equivalent to 1 (e.g. $\frac{4}{4}$) will result in an equivalent fraction (<strong>5.NF.2.5b</strong>).</td>
</tr>
<tr>
<td>Students use visual models and equations to solve problems involving the addition and subtraction of fractions, moving flexibly between the abstract and concrete representations (<strong>MP.2</strong>, <strong>MP.4</strong>).</td>
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</table>
## Unit 2

### Topic 5: Understanding the concept of multiplying fractions by fractions

In this topic, students extend their understanding of multiplying a fraction by a whole number to multiplying fractions by fractions. In previous grades, students have developed understanding of fractions as numbers. In this grade level, students develop an understanding of the connection between fractions and division. They will use this understanding to explore the relationship of multiplication and division when multiplying fractions as explained in 5.NF.2a.

### Standards

Interpret a fraction as division of the numerator by the denominator \( \left( \frac{a}{b} = a \div b \right) \). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret \( \frac{3}{4} \) as the result of dividing 3 by 4, noting that \( \frac{3}{4} \) multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

### Academic Language

- area
- denominator
- fraction
- mixed number
- numerator
- partition
- rectangular array
- unit fraction

### Students will:

- **illustrate** that the numerator represents the total amount being divided (dividend) and that the denominator represents the number of equal portions needed (divisor).
- **explain** that fractions \( \left( \frac{a}{b} \right) \) can be represented as division of the numerator by the denominator \( \left( a \div b \right) \). For example, \( \frac{5}{3} = 5 \div 3 \).
- **solve** word problems involving the division of whole numbers resulting in a fractional or mixed number quotient.

E.g., Show how \( 3 \div 7 \) can also be represented as \( \frac{3}{7} \).

Sara has 3 sub sandwiches. She would like to split the sandwiches equally between 7 people. What fraction of 1 sandwich will each person receive?

- Divide each of 3 rectangles into 7 equal parts resulting in a total of 21 one-seventh sized pieces.
- Divide the 21 one-seventh sized pieces into 7 equal groups.

- The result is 3 one-seventh sized pieces per group. \( 3 \times \frac{1}{7} = \frac{3}{7} \). So, \( 3 \div 7 = \frac{3}{7} \)
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product \((\frac{a}{b}) \times q\) as \(a\) parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((\frac{2}{3}) \times 4 = \frac{8}{3}\), and create a story context for this equation. Do the same with \((\frac{2}{3}) \times 4 = \frac{8}{15}\). (In general, \((\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}\)).

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

**Students will:**
- **use** visual fraction models (e.g., fraction strips, arrays, area models, fraction multipliers, number lines, etc.) to interpret multiplication of a whole number by a fraction
  
  E.g., \(\frac{2}{3}\) of 5 (i.e., \(\frac{2}{3} \times 5\)) is 2 parts when 5 is partitioned into 3 equal parts.

- **use** visual fraction models (e.g., fraction strips, arrays, area models, fraction multipliers, number lines, etc.) to interpret multiplication of a fraction by a fraction
  
  E.g., Extend the reasoning that since \(\frac{2}{3}\) of 5 is 2 parts when 5 is partitioned into 3 equal parts to understand that \(\frac{2}{3}\) of \(\frac{5}{6}\) (i.e., \(\frac{2}{3} \times \frac{5}{6}\)) is 2 parts when \(\frac{5}{6}\) is partitioned into 3 equal parts.

- **create** story contexts for problems involving multiplication of whole number by a fraction or multiplication of two fractions.

- **use** unit squares with fractional sides to find the area of a rectangle with fractional side lengths by tiling it, and use this tiling to prove that the area is the same as would be found by multiplying the side lengths.

- **multiply** fractional side lengths to find areas of rectangles.

- **represent** fraction products as rectangular areas.

E.g., \(\frac{2}{3} \times \frac{4}{5}\) represented using an area model that measures 1 unit by 1 unit:

![Area Model Example](image-url)
1. Make sense of problems and persevere in solving them.  
4. Model with mathematics.  
5. Use appropriate tools strategically.  

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>When labeling areas with exponential units for <strong>NF.2.4</strong>, students will use their understanding of exponents (<strong>NBT.1.2</strong>) from topic 2 to see that units(^2) is the result of multiplying units (\times) units.</td>
</tr>
<tr>
<td>Representing multiplication of fractions with visual and concrete models is fundamental to this topic in order for students to make sense of multiplying fractions by fractions (<strong>MP.1, MP.4</strong>). Students select and use a variety of tools to explore these concepts (<strong>MP.5</strong>).</td>
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</tbody>
</table>
### Topic 6: Comparing and rounding decimals

**Pacing:** November 8 – 29

In this topic students apply their understanding of comparing fractions and their understanding of place value to compare decimals.

<table>
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<tr>
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</table>
| **Read, write, and compare decimals to thousandths.**
  a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \left(\frac{1}{10}\right) + 9 \times \left(\frac{1}{100}\right) + 2 \times \left(\frac{1}{1000}\right)$.  
  b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. | **MAFS.5.NBT.1.3** base-ten numeral decimal expanded form hundredths number names tenths thousandths whole |

**Students will:**
- **read and write** decimals up to the thousandths place using base-ten numerals, number names and expanded form.
- **compare** two decimals up to the thousandths using place value and record the comparison using symbols $<$, $>$, or $=$.

**Use place value understanding to round decimals to any place.**

<table>
<thead>
<tr>
<th>Students will:</th>
<th><strong>MAFS.5.NBT.1.4</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• <strong>use</strong> place value to round decimals to any place, including the nearest whole number.</td>
<td></td>
</tr>
</tbody>
</table>

**E.g.,**

```
represent the number 3.84 on a number line
```

Possible student response:
On the number line, 3.84 is between 3 and 4. The number 3.84 is closer to 4 than to 3. So, 3.84 rounded to the nearest whole is 4.

**NOTE:** When rounding decimal numbers the final place value in the rounded number indicates where the number was rounded to; E.g., When rounding 34.632 to the nearest tenth, a result of 34.600 suggests that the number has been rounded to the thousandths place so, in order to attend to precision, the two right-most zeros should be omitted.

6. **Attend to precision.**
7. **Look for and make use of structure.**

<table>
<thead>
<tr>
<th>Topic Comments:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Students apply their understanding of the structure within the base-ten system and fraction-decimal equivalencies to precisely communicate their understanding of relative sizes of decimal numbers (MP.6, MP.7).</strong></td>
</tr>
</tbody>
</table>
### Topic 7: Interpreting multiplying fractions as scaling

In this topic students build on their work with “compare” problems in Grade 4 (4.OA.1.1) to develop a foundational understanding of multiplication as scaling. They interpret, represent, and explain situations involving multiplication of fractions. Students apply their whole number work with multiplication to develop conceptual understanding of multiplying a fraction by a fraction. Scaling is foundational for developing an understanding of ratios and proportion in future grade levels.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret multiplication as scaling (resizing), by:</td>
<td>MAFS.5.NF.2.5</td>
</tr>
<tr>
<td>a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</td>
<td>equivalent fraction factor</td>
</tr>
<tr>
<td>b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence ( \frac{a}{b} = \frac{(n\times a)}{(n\times b)} ) to the effect of multiplying ( \frac{a}{b} ) by 1.</td>
<td>fraction greater than 1 mixed number product scale factor</td>
</tr>
</tbody>
</table>

**Students will:**

- **Interpret** the relationship between the size of the factors and the size of the product without performing the actual multiplication.

**NOTE:** Whole number factors should be greater than 1,000.

E.g.,

**Example 1:**

How does the product of 3,225 x 6,000 compare to the product of 3,225 x 3,000? How do you know?

Since 3,000 is half of 6,000, the product of 3,225 x 6000 will be double or twice the product of 3,225 x 3000.

**Example 2:**

Two newspapers are comparing sales from last year.

- The Post sold 34,859 copies.
- The Tribune sold one-and-a-half times as many copies as the Post.

Write an expression which describes the number of newspapers the Tribune sold.

\[ 1\frac{1}{2} \times 34,859 \]

- **explain** why multiplying a given number by a scale factor greater than 1 (i.e., a whole number, a fraction greater than 1, or a mixed number) results in a product greater in size than the given number.
- **explain** why multiplying a given number by a scale factor less than 1 results in a product less in size than the given number.
- **explain** why multiplying a given fraction by a fraction that is equivalent to 1 as a scale factor results in a product that is equivalent in size to the given fraction and use this understanding to find equivalent fractions.

E.g., \( \frac{7}{9} \times 1 = \frac{7}{9} \times \frac{4}{4} = \frac{28}{36} \)
Solve real world problems involving multiplication of fractions and mixed numbers, e.g. by using visual fraction models or equations to represent the problem.  

**Students will:**  
- **solve** real world problems involving multiplication of fractions and mixed numbers (including multiplying mixed numbers by mixed numbers).

| MAFS.NF.2.6 |  
|---|---|

2. Reason abstractly and quantitatively.  
4. Model with mathematics.  
6. Attend to precision.  

<table>
<thead>
<tr>
<th>MAFS.K12.MP.2.1</th>
<th>MAFS.K12.MP.4.1</th>
<th>MAFS.K12.MP.6.1</th>
</tr>
</thead>
</table>

**Topic Comments:**  
In this topic, **5.NF.2.5a** and **5.NF.2.5b** involve only multiplication by fractions. Division by unit fractions will be introduced in topic 8. In **5.NF.2.6** students should have opportunities to work with all problem types, see *Common Multiplication and Division Situations Table* on pg.38.

Students reason abstractly about the size of factors and practice communicating their thinking about the relationship between scale factors and factors (**MP.2, MP.6**). They use number lines and other visual models to interpret real world situations (**MP.4**).
Topic 8: Developing the concept of dividing unit fractions

In this topic students will use their understanding of the relationship of multiplication and division to develop a conceptual understanding of division with fractions (division of a whole number by a unit fraction or a unit fraction by a whole number).

Standards

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \( \frac{1}{3} \div 4 \), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( \frac{1}{3} \div 4 = \frac{1}{12} \) because \( \frac{1}{12} \times 4 = \frac{1}{3} \).

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for \( 4 \div \frac{1}{2} \), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( 4 \div \frac{1}{2} = 20 \) because \( 20 \times \frac{1}{2} = 4 \).

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share \( \frac{1}{2} \) lb of chocolate equally? How many \( \frac{1}{3} \)-cup servings are in 2 cups of raisins?

MAFS.5.NF.2.7

Students will:

- apply an understanding of the division of whole numbers to the concept of dividing whole numbers by unit fractions (e.g., \( 4 \div \frac{1}{3} \) can be interpreted as 4 inches of ribbon being cut into \( \frac{1}{3} \) inch pieces) and dividing unit fractions by whole numbers (e.g., \( \frac{1}{3} \div 4 \) can be interpreted as sharing \( \frac{1}{3} \) of a school pizza with 4 people).

- interpret and solve problems involving the division of whole numbers by unit fractions (e.g., \( 4 \div \frac{1}{3} \) can be thought of as how many groups of \( \frac{1}{3} \) can be made from 4 wholes) using visual models and story contexts.

E.g., How many quarter cups are in 2 cups?

\[
\begin{align*}
1 \text{ cup sour cream} &= \frac{1}{4} \text{ cup} + \frac{1}{4} \text{ cup} + \frac{1}{4} \text{ cup} + \frac{1}{4} \text{ cup} \\
1 \text{ cup sour cream} &= \frac{1}{4} \text{ cup} + \frac{1}{4} \text{ cup} + \frac{1}{4} \text{ cup} + \frac{1}{4} \text{ cup} \\
\end{align*}
\]

\[
2 \div \frac{1}{4} = 8
\]

There are 8 quarter cups in 2 cups.
• **interpret and solve** problems involving the division of unit fractions by whole numbers (e.g., \(\frac{1}{3} \div 4\) can be thought of as \(\frac{1}{3}\) being divided into 4 equal parts) using visual models and story contexts.

E.g., \(\frac{1}{3} \div 4 = ?\)

By dividing each third into 4 equal parts, we show that each part is \(\frac{1}{12}\) of the whole. This means that \(\frac{1}{3} \div 4 = \frac{1}{12}\).

• **solve** real world problems involving division of unit fractions and whole numbers using fraction models and equations.

**NOTE:** Division of a fraction by a fraction is a Grade 6 standard.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

**MAFS.K12.MP.1**  **MAFS.K12.MP.2.1**

**Topic Comments:**

In this topic it is critical for students to use concrete objects or pictures to help conceptualize, create, and solve problems (MP.1, MP.2).
## Unit 3

### Topic 9: Solving problems including volume

This topic calls for students to apply their understanding of volume to real-world problems. They develop efficient strategies, including the use of formulas, to compute volumes of right rectangular prisms or other three-dimensional figures that can be broken down into non-overlapping right rectangular prisms.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base.</td>
<td>additive&lt;br&gt;associative property&lt;br&gt;B = area of base&lt;br&gt;cubic units&lt;br&gt;edge&lt;br&gt;formula&lt;br&gt;height&lt;br&gt;length&lt;br&gt;rectangular prism&lt;br&gt;unit cube&lt;br&gt;volume&lt;br&gt;width</td>
</tr>
<tr>
<td>a. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</td>
<td>MAFS.5.MD.3.5</td>
</tr>
<tr>
<td>b. Apply the formula $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</td>
<td></td>
</tr>
<tr>
<td>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of non-overlapping parts, applying this technique to solve real world problems.</td>
<td></td>
</tr>
</tbody>
</table>

**Students will:**

- **find** the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths or multiplying the height by the area of the base.

  E.g.,

  $$(3 \times 2)$$ represents the number of cubic units on the first layer<br>$$(3 \times 2) \times 5$$ represents the number of $3 \times 2$ layers<br>$$3 \times 2 + (3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2)$$ represents 5 layers with 6 cubic units each<br>$$5 \times 6 = 30$$ cubic units

- **relate** finding the product of three numbers (length, width, and height) using the associative property of multiplication to finding volume.
- **calculate** volume of rectangular prisms and cubes, with whole number edge lengths, using the formula for volume ($V = l \times w \times h$ or $V = B \times h$) in real world and mathematical problems.

  E.g., When shown an image of a rectangular prism with labeled side lengths (no cubes shown):
  1. Find the area of the base by multiplying its length by its width ($B = l \times w$).
  2. Multiply the area of the base by the height ($V = B \times h$).
- **recognize** that volume is additive by decomposing a composite solid into non-overlapping rectangular prisms to **find** the volume of the solid by finding the sum of the volumes of each of the decomposed prisms.

  E.g.,

  ![Diagram of a layered structure with volume calculation](image)

  - **solve** real world problems involving **finding** the volume of solid figures composed of two non-overlapping right rectangular prisms.

  E.g., What is the volume of water needed to fill the pool in the diagram?

  ![Diagram of a pool with volume calculation](image)

  - The deep end of the pool measures 14 ft. × 10 ft. × 5 ft. making the volume 700 ft³.
  - The shallow end of the pool measures 6 ft. × 5 ft. × 5 ft. making the volume 150 ft³.
  - $700 \text{ ft}^3 + 150 \text{ ft}^3 = 850 \text{ ft}^3$

**NOTE:** Figures may contain no more than two non-overlapping prisms – non-overlapping means that two prisms may share a face, but they do not share the same volume. Labels may include cubic units (i.e. cubic centimeters, cubic feet, etc.) or exponential units (i.e., cm³, ft³, etc.).
<table>
<thead>
<tr>
<th>Topic Comments:</th>
</tr>
</thead>
<tbody>
<tr>
<td>When labeling volumes with exponential units, students will use their understanding of exponents (NBT.1.2) from topic 2 to see that units(^3) is the result of multiplying units x units x units. Students pack the figures with unit cubes (MP.5) and connect this structure to multiplicative reasoning (MP.7). They solve problems by applying the generalized formulas (MP.8).</td>
</tr>
</tbody>
</table>
# Topic 10: Performing operations with decimals

Measurement is used in this topic as a context for operations with decimals. In Grade 4, students converted measurements given in a larger unit to a smaller unit. Students’ previous experiences with decimal fractions and fraction computations are applied here to provide multiple ways of thinking about operations with decimals. Students can use their understanding of decimal-fraction equivalencies, concrete or visual models, and place value to reason about decimal quantities and operations.

## Standards

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
<td>MAFS.5.NBT.2.7 conversion, convert customary units</td>
</tr>
</tbody>
</table>

**Students will:**
- **add** decimals to hundredths, using concrete models, drawings, strategies based on place value, and/or properties of operations.
- **subtract** decimals to hundredths, using concrete models, drawings, strategies based on place value, properties of operations and/or the relationship between addition and subtraction.
- **multiply** decimals using rectangular arrays, area models, and/or an understanding of place value and properties of operations.
- **divide** decimals using rectangular arrays, area models, an understanding of place value and properties of operations, and/or the relationship between multiplication and division.
- **represent and explain** strategies and reasoning used to solve problems including decimals.

E.g., Use a model to solve $3 - 0.6$.

![Model for subtraction](image)

**NOTE:** This standard requires students to extend the models and strategies previously developed for conceptual understanding of operations with whole numbers. Using the standard algorithms for adding, subtracting, multiplying, and dividing decimals is a Grade 6 standard.

Convert among different-sized standard measurement units (i.e., km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec) within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

**NOTE:** This standard has been amended in Florida to include specific units of measure.

**Students will:**

**NOTE:** Students need to be provided with the Grade 5 FSA Mathematics Reference Sheet (see pg. 39) in order to practice using the tool.

- **convert** units (required units are listed on Grade 5 FSA Mathematics Reference Sheet) within the same system (customary to customary and metric to metric).
- **apply** knowledge of length, weight, mass, and time to solve multi-step word problems using measurement conversions.

**NOTE:** Measurement values may be fractional, decimal, or whole number values.
| 2. Reason abstractly and quantitatively. | MAFS.K12.MP.2.1 |
| 3. Construct viable arguments and critique the reasoning of others. | MAFS.K12.MP.3.1 |

**Topic Comments:**

5.MD.1.1 provides measurement conversion as a context for not only working with decimals but a deeper understanding for place value and the connection to the metric system.

Instead of just computing answers, students reason about both the relationship between fraction and decimal operations and the relationship between whole number computation and fractional/decimal computation (MP.2, MP.3).
### Topic 11: Classifying two-dimensional geometric figures

In this topic the emphasis is on the hierarchical relationship among 2-dimensional geometric figures. Students have had previous experience classifying shapes using defining attributes, and this topic extends this concept to set a foundation for understanding the extension of properties from category to subcategory.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <em>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</em></td>
<td>MAFS.5.G.2.3 defining attribute category classify congruent parallel perpendicular polygons right angles subcategory two-dimensional figures Venn diagram</td>
</tr>
</tbody>
</table>

**Students will:**

- **categorize** two-dimensional figures (i.e., triangle, quadrilateral, rectangle, square, rhombus, trapezoid) according to their defined attributes.

  **NOTE:** Geometric (defining) attributes include number and properties of sides (i.e., parallel, perpendicular, congruent) and number and properties of angles (i.e., type of angle, measurement of angle, congruency of angles).

- **relate** certain categories of two-dimensional figures as subcategories of other categories

  E.g., rectangles are a subcategory of parallelograms because they are both quadrilaterals with two pairs of sides that are parallel and congruent, therefore all rectangles are also parallelograms.

- **explain** that the attributes belonging to one category of two-dimensional figures also belong to all subcategories of that category.

  E.g., Squares are a subcategory of rectangles, therefore if rectangles have four right angles and two pairs of sides that are parallel and congruent, it can be inferred that squares have four right angles and two pairs of sides that are parallel and congruent.
Classify and organize two-dimensional figures into Venn diagrams based on the attributes of the figures.

**NOTE:** This standard has been amended in Florida to include Venn diagrams.

<table>
<thead>
<tr>
<th>MAFS.5.G.2.4</th>
</tr>
</thead>
</table>

**Students will:**

- **classify** two-dimensional figures (i.e., triangle, quadrilateral, rectangle, square, rhombus, trapezoid) based on defining attributes.
- **organize** two-dimensional figures into a Venn diagram based on defining attributes.

**E.g.,**

![Venn diagram showing classification of quadrilaterals](image)

**NOTE:** Use the exclusive definition of trapezoid: **exactly** one set of parallel sides.

<table>
<thead>
<tr>
<th>Topic Comments:</th>
</tr>
</thead>
</table>

Students make use of structure to build a logical progression of statements and explore hierarchical relationships among 2-dimensional shapes (MP.3, MP.7).

<table>
<thead>
<tr>
<th>MAFS.K12.MP.3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.K12.MP.3.1</td>
</tr>
</tbody>
</table>
## Topic 12: Solving problems with fractional quantities

In this topic students use data and other contexts to solve real world problems involving fractional computations. All of the problem types in the Common Addition and Subtraction and Multiplication and Division Situations Tables (see pages 55 and 56) should be addressed in this topic.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make a line plot to display a data set of measurements in fractions of a unit ((\frac{1}{2}, \frac{1}{4}, \frac{1}{8})). Use operations on fractions for this grade to solve problems involving information presented in line plots. <em>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</em></td>
<td>MAFS.5.MD.2.2</td>
</tr>
</tbody>
</table>

**Students will:**
- *create* a line plot recording measurement data including fraction units of halves, quarters, and eighths.
- *use* the measurement data on a line plot to solve multistep problems involving fractions and draw conclusions about the data.

![Line plot example](image)

**NOTE:** Since students worked with categorical data and bar graphs in Grades 2 & 3, a student might find it natural to summarize a measurement data set by viewing it in terms of categories—the categories in question being the three distinct length values which appear on the number line above. For example, the student might want to say that there are two observations in the “category” of \(8\frac{1}{4}\) inches. However, it is important to recognize that \(8\frac{1}{4}\) inches is not a category like “blue, yellow or red” Unlike these colors, \(8\frac{1}{4}\) inches is a numerical value with a measurement unit. That difference is why the data in this table are called measurement data and presented on a line plot rather than a bar graph. A display of measurement data must present the measured values with their appropriate magnitudes and spacing on the number line of the line plot.

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result \(\frac{1}{5} + \frac{1}{2} = \frac{3}{7}\), by observing that \(\frac{3}{7} < \frac{1}{2}\).*

**Students will:**
- *use* data to *solve* real world problems involving addition and subtraction of fractions with like and unlike denominators.

Solve real world problems involving multiplication of fractions and mixed numbers, e.g. by using visual fraction models or equations to represent the problem.

**Students will:**
- *use* data to *solve* real world problems involving multiplication of fractions and mixed numbers.
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$-lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?  

<table>
<thead>
<tr>
<th><strong>Students will:</strong></th>
<th><strong>MAFS.5.NF.2.7</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• use data to solve real world problems involving division of whole numbers by unit fractions using fraction models and equations.</td>
<td></td>
</tr>
</tbody>
</table>

2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.

| **Topic Comments:** | **MAFS.K12.MP.2.1**
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.2.2 is included here so measurement line plots can be used as a context for students to apply fraction computation strategies.</td>
<td>MAFS.K12.MP.5.1</td>
</tr>
</tbody>
</table>

Students use line plots and other tools/technology to reason about problem situations (MP.5). Students attend to the underlying meaning of the quantities and operations when solving problems rather than just how to compute answers (MP.2).
# Unit 4

## Topic 13: Representing algebraic thinking

In this topic students explore numerical expressions more formally to represent and interpret calculations involving whole numbers, fractions, and decimals. They apply their understanding of the different algebraic properties of operations and explain the relationships between the quantities with the written expressions. This topic includes opportunities to both evaluate expressions and reason about expressions without calculating a solution. This is foundational for further work with number in later grades.

### Standards

<table>
<thead>
<tr>
<th>Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</th>
<th>MAFS.5.OA.1.1</th>
</tr>
</thead>
</table>

**Students will:**

- **evaluate** expressions which include parentheses, brackets, or braces by performing operations in the conventional order (operations inside grouping symbols, multiplication and division from left to right, then addition and subtraction from left to right).

  \[
  \frac{1}{4} \times [2 + 6 \times 3] - 4 \\
  \frac{1}{4} \times [2 + 18] - 4 \\
  \frac{1}{4} \times 20 - 4 \\
  5 - 4 \\
  1
  \]

  In which step does a mistake first appear?

  \[
  \frac{1}{2} \times \{6 \times 1 + 7\} + 11 \\
  \]

  Step 1: \( \frac{1}{2} \times (6 \times 8) + 11 \)

  Step 2: \( \frac{1}{2} \times 48 + 11 \)

  Step 3: \( 24 + 11 \)

  Step 4: 35

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as 2 \( \times (8 + 7) \). Recognize that 3 \( \times (18932 + 921) \) as three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

<table>
<thead>
<tr>
<th>MAFS.5.OA.1.2</th>
</tr>
</thead>
</table>

**Students will:**

- **apply** an understanding of operations and grouping symbols to **write** numerical expressions.

- **apply** an understanding of operations and grouping symbols to **interpret** the meaning of numerical expressions without evaluating (i.e., calculating) them.

6. **Attend to precision.**

<table>
<thead>
<tr>
<th>MAFS.K12.MP.6.1</th>
</tr>
</thead>
</table>

### Academic Language

- braces
- brackets
- evaluate
- numerical expression
- operation
- parentheses
- value

### Topic Comments:

The expressions described in **5.OA.1.1** include the use of parentheses, brackets, or braces, but should not contain nested grouping symbols. Teachers should not use mnemonic devices such as PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) for remembering Order of Operations because they have the potential to cause students to focus on the device, rather than on the underlying mathematical meaning of an expression. This can lead to serious misconceptions in future study.

The expressions described in **5.OA.1.2** should be no more complex than the expressions one finds in an application of the associative or distributive property.

Students discuss the meaning of symbols and interpret numerical expressions precisely (MP.6).
**Topic 14: Exploring the coordinate plane**

In this topic students are introduced to the coordinate plane, applying their knowledge of the number line to understand the relationship of the two dimensions of a point in the coordinate plane. Students connect their work with numerical patterns to form ordered pairs and graph these ordered pairs in the first quadrant of a coordinate plane. Students use this model to make sense of and explain the relationships within the numerical patterns they generate. This prepares students for future work with functions and proportional relationships in the middle grades.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate two numerical patterns using two given rules. Identify apparent</td>
<td>axes, coordinate plane, coordinates, corresponding terms, horizontal plot, rule, ordered pairs, origin, point, quadrant, sequence, vertical, horizontal, x-axis, y-axis, x- and y-coordinates.</td>
</tr>
<tr>
<td>relationships between corresponding terms. Form ordered pairs consisting</td>
<td>MAFS.5.OA.2.3</td>
</tr>
<tr>
<td>of corresponding terms from the two patterns, and graph the ordered</td>
<td></td>
</tr>
<tr>
<td>pairs on a coordinate plane. *For example, given the rule “Add 3” and</td>
<td></td>
</tr>
<tr>
<td>the starting number 0, and given the rule “Add 6” and the starting</td>
<td></td>
</tr>
<tr>
<td>number 0, generate terms in the resulting sequences, and observe that</td>
<td></td>
</tr>
<tr>
<td>the terms in one sequence are twice the corresponding terms in the other</td>
<td></td>
</tr>
<tr>
<td>sequence. Explain informally why this is so.</td>
<td></td>
</tr>
</tbody>
</table>

**Students will:**
- **generate** two numerical patterns using two given rules.
- **explain** the relationship between the two numerical patterns by comparing the relationship between each of the corresponding terms from each pattern.
- **form** ordered pairs out of corresponding terms from each pattern.
- **graph** the ordered pairs on a coordinate plane.

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

**Students will:**
- **draw** a coordinate plane with two intersecting perpendicular lines.
- **identify** the intersection as the origin and the point where 0 lies on each of the number lines.
- **label** the horizontal axis as the x-axis, and the vertical axis as the y-axis.
- **explain** that when plotting points on a coordinate plane the first number in an ordered pair (the x-coordinate) indicates how far to travel from the origin in the direction of the x-axis and that the second number (y-coordinate) indicates how far to travel in the direction of the y-axis.

*E.g.,* When graphing the ordered pair (3,2), 3 is the x-coordinate so you move 3 units from 0 on the x-axis and then, since 2 is the y-coordinate, you move up 2 units in the direction of the y-axis.

**NOTE:** Students are only expected to utilize the first quadrant of the coordinate plane which includes only positive numbers.
Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

**Students will:**
- **represent** real world and mathematical problems by graphing points in the first quadrant of the coordinate plane.
  E.g., Students plan to draw a symmetric figure in which they plot coordinates that are then connected by line segments.
- **interpret** coordinate values of points in the context of the situation.

**NOTE:** Students need to be able to use directional language or a compass rose and cardinal directions.
(i.e., North, South, East, or West)

6. Attend to precision.
7. Look for and make use of structure.

**Topic Comments:**
Students precisely describe the coordinates of points and the relationship of the coordinate plane to the number line (MP.6). Students both generate and identify relationships in numerical patterns, using the coordinate plane as a way of representing these relationships and patterns (MP.7).
<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluent multi-digit whole numbers using the standard algorithm.</td>
<td>MAFS.5.NBT.2.5</td>
</tr>
<tr>
<td><strong>Students will:</strong></td>
<td></td>
</tr>
<tr>
<td>• multiply multi-digit whole numbers using the standard algorithm.</td>
<td></td>
</tr>
<tr>
<td><strong>NOTE:</strong> Multiplication should not exceed 5 digits by 2 digits. Computational fluency is defined as accuracy, efficiency, and flexibility.</td>
<td></td>
</tr>
<tr>
<td>Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
<td>MAFS.5.NBT.2.6</td>
</tr>
<tr>
<td><strong>Students will:</strong></td>
<td></td>
</tr>
<tr>
<td>• solve division of a multi-digit dividend by a two-digit divisor using strategies based on place value, properties of operations, and/or the relationship between multiplication and division.</td>
<td></td>
</tr>
<tr>
<td>• illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
<td></td>
</tr>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>MAFS.K12.MP.1.1</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>MAFS.K12.MP.8.1</td>
</tr>
</tbody>
</table>

**Topic Comments:**

5.NBT.2.6 is a milestone along the way to reaching fluency with the standard algorithm for division in Grade 6 (6.NS.2.2). Students use efficient strategies and look for shortcuts to multiply and divide whole numbers with accuracy (MP.1, MP.8).
## Topic 16: Revisiting problem solving with fractions

This is a culminating topic in which students apply their conceptual understanding from Grades 3-4 and previous Grade 5 topics to a variety of non-routine problem solving situations involving grade-level appropriate operations with fractions. All standards in this topic have been addressed in prior topics. In grade 6, students will finalize their exploration of fraction operations with dividing fractions by fractions using visual models and equations.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. <em>For example, recognize an incorrect result</em> ( \frac{5}{2} \times \frac{1}{2} = \frac{3}{7} ), by observing that ( \frac{7}{7} &lt; \frac{1}{2} ).</td>
<td>MAFS.5.NF.1.2</td>
</tr>
<tr>
<td>Students will:</td>
<td>denominator</td>
</tr>
<tr>
<td>- solve word problems involving addition and subtraction of fractions with like and unlike denominators.</td>
<td>dividend</td>
</tr>
<tr>
<td>- use benchmark fractions and number sense of fractions to estimate and assess reasonableness of answers.</td>
<td>divisor</td>
</tr>
<tr>
<td>Solve real world problems involving multiplication of fractions and mixed numbers, e.g. by using visual fraction models or equations to represent the problem.</td>
<td>equivalent fraction</td>
</tr>
<tr>
<td>Students will:</td>
<td>factor</td>
</tr>
<tr>
<td>- solve real world problems involving multiplication of fractions and mixed numbers.</td>
<td>fraction greater than 1</td>
</tr>
</tbody>
</table>

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share \( \frac{1}{2} \)-lb of chocolate equally? How many \( \frac{1}{3} \)-cup servings are in 2 cups of raisins? |

Students will:

- solve real world problems involving division of unit fractions and whole numbers using fraction models and equations.

**NOTE:** Division of a fraction by a fraction is a Grade 6 standard.

1. Make sense of problems and persevere in solving them.  
4. Model with mathematics.

### Topic Comments:

5.NF.2.7 is a prerequisite standard for 6.NS.1.1.

Students use visual models and equations to interpret real world situations involving fractions. (MP.1, MP.4).
Critical Areas for Mathematics in Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (i.e., unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
## Grade 5 Major, Supporting, and Additional Work

<table>
<thead>
<tr>
<th>Topic</th>
<th>Title</th>
<th>Major Work</th>
<th>Supporting Work</th>
<th>Additional Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Understanding volume</td>
<td>5.MD.3.3, 5.MD.3.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Expanding understanding of place value of decimals</td>
<td>5.NBT.1.1, 5.NBT.1.2, 5.NBT.1.3 (a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Developing multiplication and division strategies</td>
<td>5.NBT.2.5, 5.NBT.2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Using equivalency to add and subtract fractions with unlike denominators</td>
<td>5.NF.1.1, 5.NF.1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Understanding the concept of multiplying fractions by fractions</td>
<td>5.NF.2.3, 5.NF.2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Comparing and rounding decimals</td>
<td>5.NBT.1.3, 5.NBT.1.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Interpreting multiplying fractions as scaling</td>
<td>5.NF.2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Developing the concept of dividing unit fractions</td>
<td>5.NF.2.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Solving problems involving volume</td>
<td>5.MD.3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Performing operations with decimals</td>
<td>5.NBT.2.7, 5.MD.1.1</td>
<td></td>
<td>5.G.2.3, 5.G.2.4</td>
</tr>
<tr>
<td>11</td>
<td>Classifying two-dimensional geometric figures</td>
<td></td>
<td>5.G.2.3, 5.G.2.4</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Solving problems with fractional quantities</td>
<td>5.NF.1.2, 5.NF.2.6, 5.NF.2.7</td>
<td></td>
<td>5.MD.2.2</td>
</tr>
<tr>
<td>13</td>
<td>Representing algebraic thinking</td>
<td></td>
<td></td>
<td>5.OA.1.1, 5.OA.1.2</td>
</tr>
<tr>
<td>14</td>
<td>Exploring the coordinate plane</td>
<td></td>
<td></td>
<td>5.OA.2.3, 5.G.1.1, 5.G.1.2</td>
</tr>
<tr>
<td>15</td>
<td>Revisiting multiplication and division with whole numbers</td>
<td>5.NBT.2.5, 5.NBT.2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Revisiting problem solving with fractions</td>
<td>5.NF.1.2, 5.NF.2.6, 5.NF.2.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Standards for Mathematical Practice

Grade 5 students will:

1. Make sense of problems and persevere in solving them. (SMP.1)
   Mathematically proficient students in Grade 5 solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”

2. Reason abstractly and quantitatively. (SMP.2)
   Mathematically proficient students in Grade 5 recognize that a number represents a specific quantity. They extend this understanding from whole numbers to work with fractions and decimals. This involves two processes: decontextualizing and contextualizing. Grade 5 students decontextualize by taking a real-world problem and writing and solving equations based on the word problem. For example, consider the task, “There are 2 2/3 of a yard of rope in the shed. If a total of 4 1/6 yard is needed for a project, how much more rope is needed?” Students decontextualize the problem by writing the equation $4 \frac{1}{6} - 2 \frac{2}{3} = ___$ and then solving it. Further, students contextualize the problem after they find the answer, by reasoning that $1 \frac{3}{6}$ or $1 \frac{1}{2}$ yards of rope is the amount needed. Further, Grade 5 students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. (SMP.3)
   Mathematically proficient students in Grade 5 construct arguments using representations, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking through discussions or written responses.

4. Model with mathematics. (SMP.4)
   Mathematically proficient students in Grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

5. Use appropriate tools strategically. (SMP.5)
   Mathematically proficient students in Grade 5 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

6. Attend to precision. (SMP.6)
   Mathematically proficient students in Grade 5 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.

7. Look for and make use of structure. (SMP.7)
   Mathematically proficient students in Grade 5 look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

8. Look for and express regularity in repeated reasoning. (SMP.8)
   Mathematically proficient students in Grade 5 use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.
## Common Addition and Subtraction Situations Table

<table>
<thead>
<tr>
<th>Add to</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?</td>
<td>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?</td>
<td>Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?</td>
</tr>
<tr>
<td>(2 + 3 = ?)</td>
<td>(2 + ? = 5)</td>
<td>(? + 3 = 5)</td>
</tr>
<tr>
<td>Take from</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Five apples were on the table. I ate two apples. How many apples are on the table now?</td>
<td>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?</td>
<td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?</td>
</tr>
<tr>
<td>(5 - 2 = ?)</td>
<td>(5 - ? = 3)</td>
<td>(? - 2 = 3)</td>
</tr>
<tr>
<td>Total Unknown</td>
<td>Both Addends Unknown(^1)</td>
<td>Addend Unknown(^2)</td>
</tr>
<tr>
<td>Three red apples and two green apples are on the table. How many apples are on the table?</td>
<td>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?</td>
<td>Five apples are on the table. Three are red and the rest are green. How many apples are green?</td>
</tr>
<tr>
<td>(3 + 2 = ?)</td>
<td>(5 = ? + ?)</td>
<td>(3 + ? = 5)</td>
</tr>
<tr>
<td>(5 = 0 + 5, 5 = 5 + 0)</td>
<td>(5 = 1 + 4, 5 = 4 + 1)</td>
<td>(5 - 3 = ?)</td>
</tr>
<tr>
<td>(5 = 2 + 3, 5 = 3 + 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference Unknown</td>
<td>Bigger Unknown</td>
<td>Smaller Unknown</td>
</tr>
<tr>
<td>&quot;How many more?&quot; version: Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?</td>
<td>&quot;More&quot; version suggests operation: Julie has 3 more apples than Lucy. Lucy has two apples. How many apples does Julie have?</td>
<td>&quot;Fewer&quot; version suggests operation: Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?</td>
</tr>
<tr>
<td>&quot;How many fewer?&quot; version: Lucy has two apples. Julie has five apples. How may fewer apples does Lucy have than Julie?</td>
<td>&quot;Fewer&quot; version suggests wrong operation: Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have?</td>
<td>&quot;More&quot; version suggests wrong operation: Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?</td>
</tr>
<tr>
<td>(2 + ? = 5)</td>
<td>(2 + 3 = ?)</td>
<td>(5 - 3 = ?)</td>
</tr>
<tr>
<td>(5 - 2 = ?)</td>
<td>(3 + 2 = ?)</td>
<td>(? + 3 = 5)</td>
</tr>
</tbody>
</table>

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity, National Research Council, 2009, pp. 32–33.

\(^1\) This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that \(=\) does not always mean “makes” or “results in” but always means “is the same number as.” Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all the decompositions of a number and reflecting on the patterns involved.

\(^2\) Either addend can be unknown; both variations should be included.
### Common Multiplication and Division Situations Table

<table>
<thead>
<tr>
<th>Equal Groups</th>
<th>Unknown Product</th>
<th>Group Size Unknown</th>
<th>Number of Groups Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>There are 3 bags with 6 plums in each bag. How many plums are there in all?</td>
<td>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</td>
<td>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</td>
</tr>
<tr>
<td>Arrays², Area³</td>
<td>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</td>
<td>Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</td>
<td>Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</td>
</tr>
<tr>
<td>Compare⁴</td>
<td>There are 3 rows of apples with 6 apples in each row. How many apples are there?</td>
<td>Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</td>
<td>Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</td>
</tr>
<tr>
<td></td>
<td>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</td>
<td>A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does the blue hat cost?</td>
<td>A red hat was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</td>
</tr>
<tr>
<td></td>
<td>A blue hat costs $6. A red hat cost 3 times as much as the blue hat. How much does the red hat cost?</td>
<td>A red hat costs $18 and that is 3 times as much as the blue hat costs. How much does the blue hat cost?</td>
<td>Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</td>
</tr>
<tr>
<td></td>
<td>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?</td>
<td>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>If 18 plums are arranged into equal rows of 6 apples, how many rows will there be?</td>
<td>A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?</td>
<td></td>
</tr>
</tbody>
</table>

1. The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
2. The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
3. Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
4. Multiplicative Compare problems appear first in Grade 4, with whole-number values for A, B, and C, and with the “times as much” language in the table. In Grade 5, unit fractions language such as “one third as much” may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., “A red hat costs A times as much as the blue hat” results in the same comparison as “A blue hat costs \( \frac{1}{a} \) times as much as the red hat,” but has a different subject.
Grade 5 FSA Mathematics Reference Sheet

Customary Conversions

1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet
1 mile = 1,760 yards

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 pound = 16 ounces
1 ton = 2,000 pounds

Metric Conversions

1 meter = 100 centimeters
1 meter = 1000 millimeters
1 kilometer = 1000 meters

1 liter = 1000 milliliters

1 gram = 1000 milligrams
1 kilogram = 1000 grams

Time Conversions

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 days
1 year = 52 weeks